

Exotic Power Sources

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Outline

Geothermal Power

Ocean Thermal Energy Conversion

The Physics of Tapping into Water Power

Tidal Barrage Power

Tidal Stream Power

Wave Power

Floating Photovoltaic Farms

Wind Power Balloons & Kites

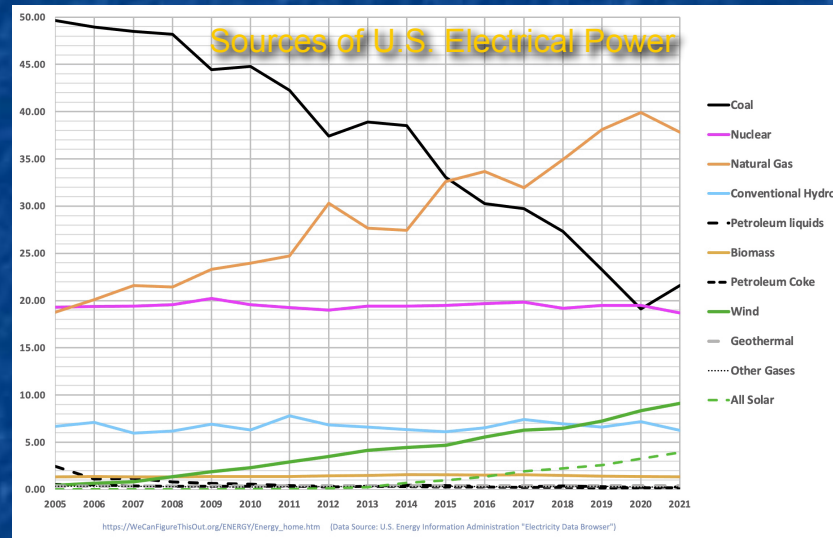
Solar Power Satellites

Fusion Power

Exotic Power Sources

This note set is about power technologies missing from this figure ¹

(plus one that does appear, but only as the figure's flat gray baseline)



But this note set is also about technologies capturing major popular press attention, emerging technologies upon which many place their hopes for a greener world. In response I will try to not only introduce these technologies but to use basic science to estimate how much available power each technology is trying to tap into, which should provide a basis for judging that technology's ultimate potential.

1) From my note set; *U.S. Energy Production & Consumption* ([pptx](#) / [pdf](#) / [key](#))

Geothermal Power

Geothermal Power is actually driven by radioactivity

Specifically, the fission decay of trace radioactive elements in the earth's core

That's why, 4.5 billion years after its formation, the earth's core **remains** molten

This heat drifts (via convection & conduction) upward towards the earth's surface

Where it is transferred **to** the atmosphere (by conduction)

or **through** the atmosphere (by weak infrared radiation)

The net result is a steep drop in temperature towards the earth's surface:

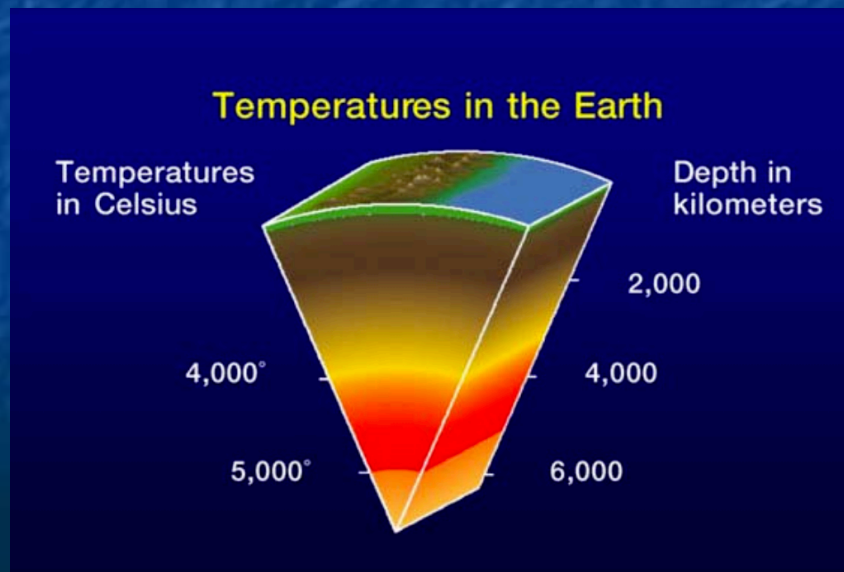


Figure:

Geothermal 101: Basics of Geothermal Energy Production and Use:

https://geothermalcommunities.eu/assets/elearning/7.15.geo_101.pdf

It's called the **Geothermal Gradient**:

Away from crustal irregularities (e.g., plate tectonic boundaries or volcanism),
the **Geothermal Gradient** is typically **25-30 °C / km of depth** ¹

As a result, while near-surface ground is typically a cool **10-15 °C** (50-60 °F),

Two kilometers below, the temperature is 60-75 °C (140-170 °F)

Hot enough to boil many organic fluids into high-pressure vapor

Three kilometers below, the temperature is 85-105 °C (185-220 °F)

Hot enough to begin boiling water into high pressure steam

By piping those organic fluids or water down into the ground 2-3 km deep

(or by heating *them* via *another* fluid piped to and from those depths)

The resulting high-pressure vapor / steam can drive turbine electrical generators

yielding **Geothermal Electrical Power + Heat** (for local factories / homes)

1) See for example Wikipedia's webpage: https://en.wikipedia.org/wiki/Geothermal_gradient

Examples of large present day Geothermal Power Plants?

They naturally favor sites where the Geothermal Gradient is exceptionally steep

Which occurs near those "crustal irregularities" where plate tectonics has drawn hot magma reservoirs unusually close to the surface

As in plate-tectonic-boundary-straddling **Iceland**: Where **25% of power** is Geothermal ¹

And in "Ring of Fire" (tectonic-plate-subduction-overlying) **Northern California**:

Where the **15** "Geysers" power plants together produce up to **725 MW** ²

Iceland's Nesjavellir Power Plant:

https://en.wikipedia.org/wiki/File:NesjavellirPowerPlant_edit2.jpg



One of the California "Geysers" Plants:

<https://pubs.usgs.gov/gip/dynamic/geothermal.html>

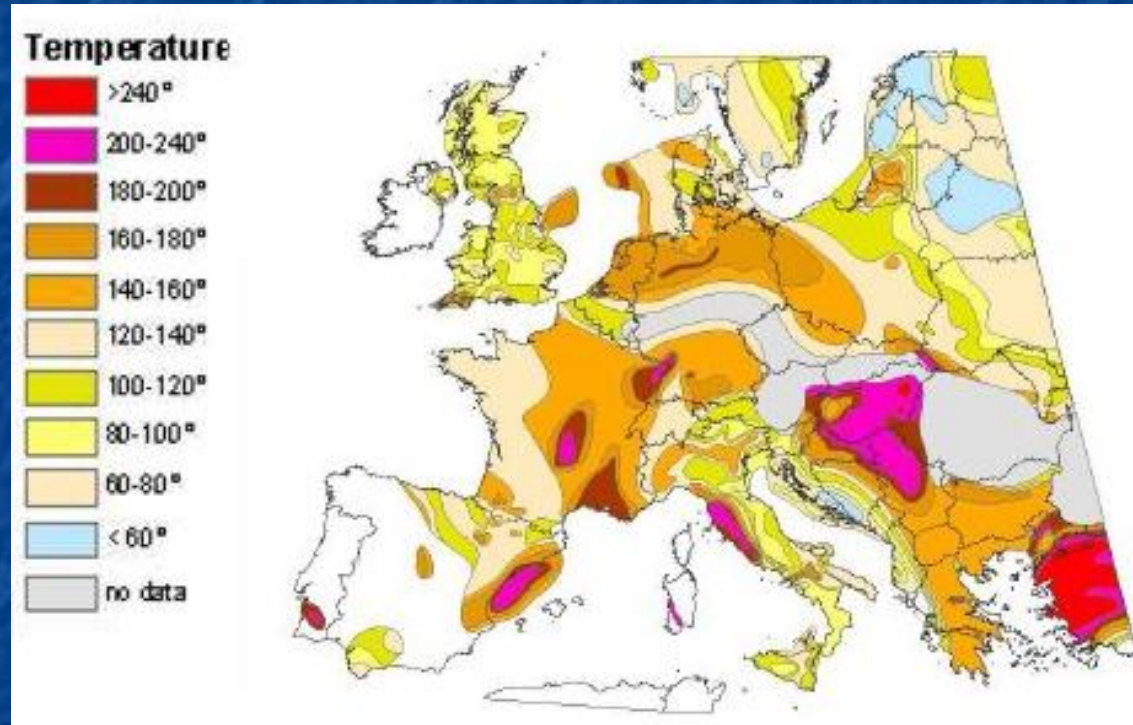


1) Orkustofnun – National Power Authority: <https://nea.is/geothermal/>

2) www.geysers.com/geothermal.aspx

What are the most promising European locations?

Extrapolated ground temperatures at 5 km depth have been plotted as:



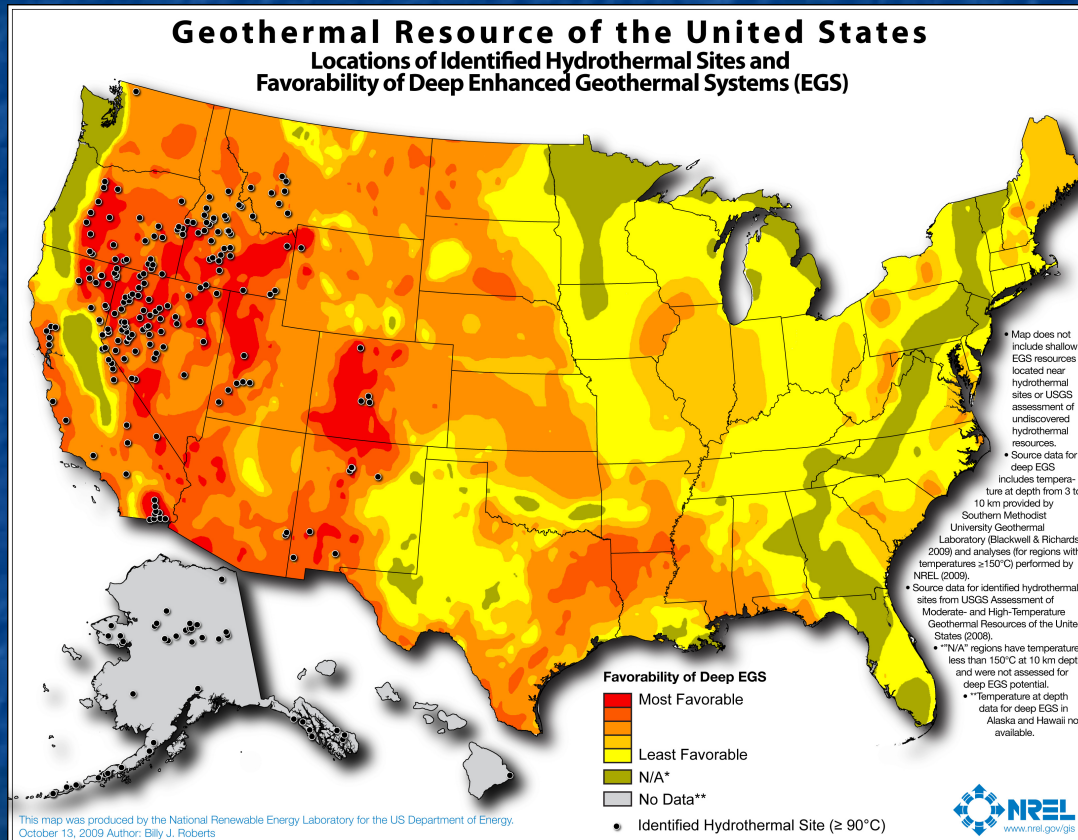
Europe's **hottest** Geothermal prospects?

Turkey, eastern Spain, Balkan mountains, some pockets in France

Or the most promising U.S. locations?

"Favorability" according to a U.S. National Renewable Energy Lab (NREL):

Small print: "Least Favorable" **yellow** sites have ground $< 150^{\circ}\text{C}$ at 10 km depth



Source: http://www.nrel.gov/gis/images/geothermal_resource2009-final.jpg

The U.S.'s **hottest** Geothermal prospects?

In lower 48's west / northwest

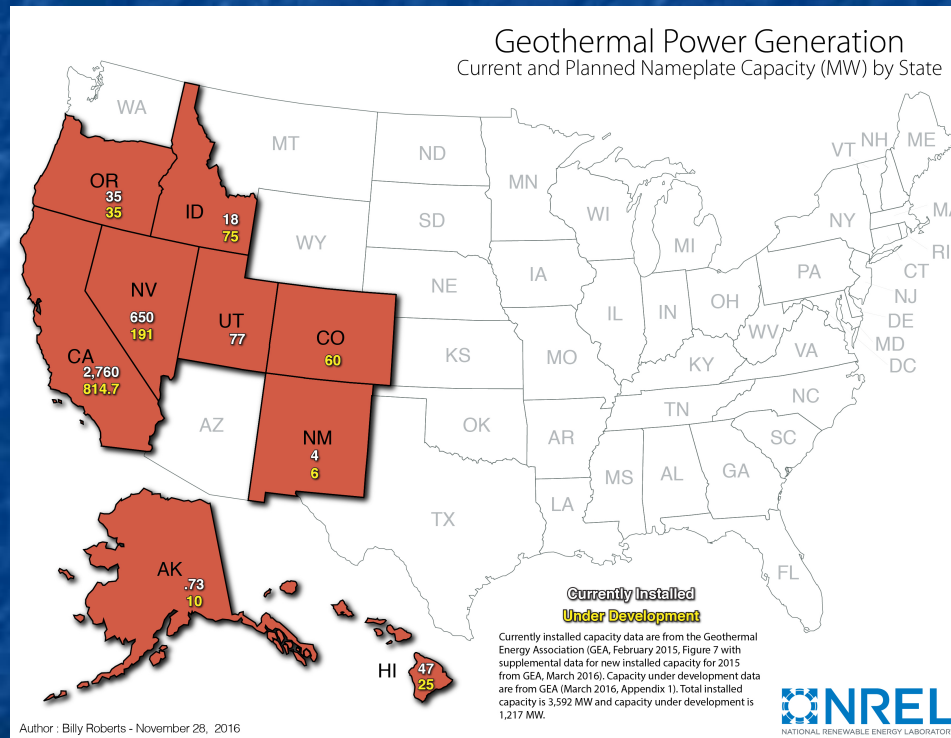
(With data needed Alaska & Hawaii - where I'd expect some excellent prospects!)

The Good News: Plants built / being built in 48's W / NW + AK + HI:

A different NREL map of "Current and Planned" Geothermal Plants by state giving:

MW Geothermal Capacity of existing state plants (numbers in **white**)

MW Geothermal Capacity of planned state plants (numbers in **yellow**)



Source: https://www.nrel.gov/gis/images/2016-11-28_Geothermal_Capacity.jpg

The Bad News:

TOTAL existing capacity is 3592 MW (comparable to ~ 6 "typical" U.S. power plants)

TOTAL excluding California is 832 MW (comparable to ~ 1 "typical" U.S. power plant)

Those Geothermal power capacity numbers are discouraging

Especially when you consider Geothermal's claimed **green energy** qualities:

- It employs simple, well known, low risk, technologies (described later)
- It produces constant 24/7 ("base load") power that requires
neither supplemental energy storage technology nor large reservoirs
- It emits no greenhouse combustion gases
- It seems to neither produce nor release dangerous wastes
- Its plants require minimal ground surface area
- Their "fuel" is drawn entirely from the hot ground immediately below them

But time-averaged U.S. power consumption is $\sim 1/2$ TW (= 500,000 MW) ¹

To supply even **10%** of that power, from the previous slide's numbers:

Geothermal capacity would have to increase by $\sim 5 \times 10^5$ MW / 3592 MW

= 140X

1) That 1/2 TW number is discussed in my note set: [U.S. Energy Production & Consumption](#) ([pptx](#) / [pdf](#) / [key](#)) note set:

The proposed solution? EGS

Which stands for **Enhanced / Engineered Geothermal Systems**

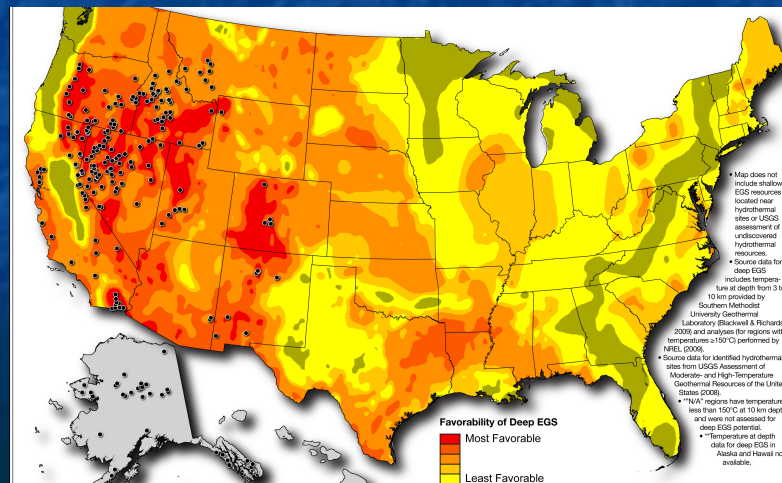
From **Geothermal 101: Basics of Geothermal Energy Production and Use**: ¹

"Although the deeper crust and interior of the Earth is universally hot, it lacks two of the three ingredients required for a naturally occurring geothermal reservoir:

Water and interconnected open volume for water movement.

Producing electricity from this naturally occurring hot, but relatively dry rock requires enhancing the potential reservoir **by fracturing, pumping water into and out of the hot rock, and directing the hot water to a geothermal power plant.**"

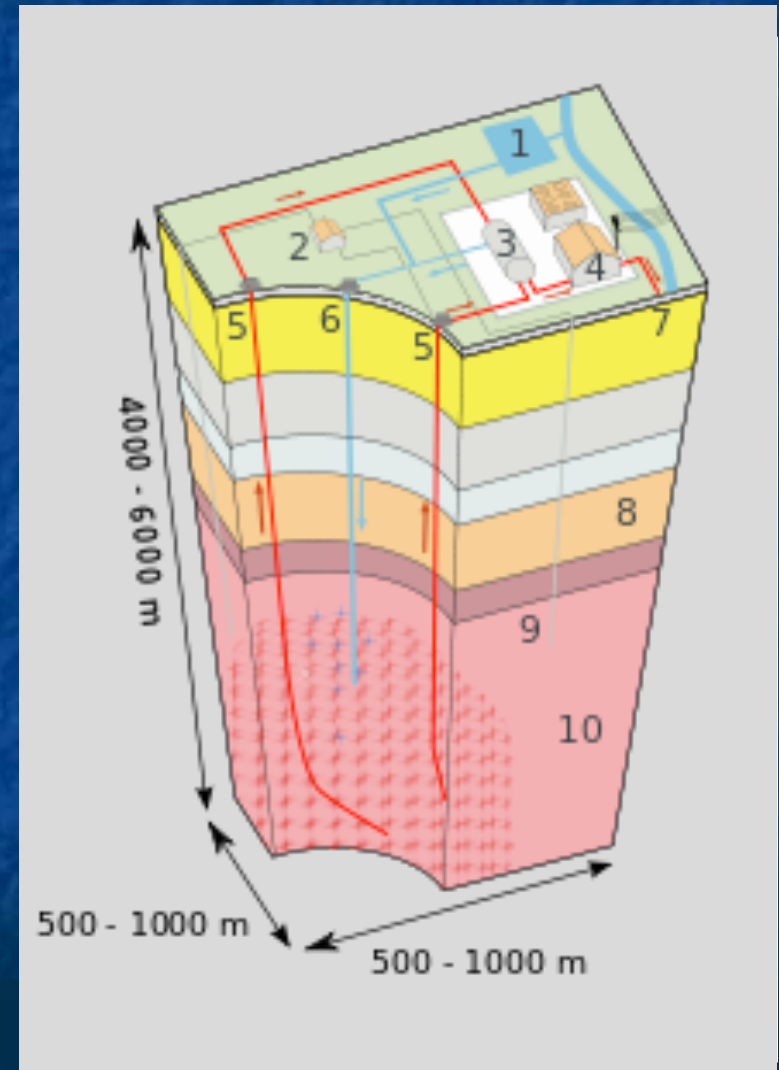
EGS might allow expansion of Geothermal out from **Red** into **Orange** U.S. areas:



What would an EGS Geothermal Plant look like?

In this Wikipedia figure, EGS plant components are:

- 1) (Surface) Reservoir
- 2) Pump house
- 3) Heat exchanger
- 4) Turbine Hall
- 5) Production Well
- 6) Injection Well
- 7) Hot Water to District Heating
- 8) Porous Sediments
- 9) Observation Well
- 10) Crystalline Bedrock



What are all those components doing?

Pump house (2) pushes supply water down into the
Injection Well (6) from which it then flows through deep extremely hot
Porous Sediments (8) causing the water to superheat and/or boil, then exit via the
Production Well (5) from which it is then routed to a
Heat exchanger (3) boiling mineral-free water with that clean steam going to the
Turbine Hall (4), with output possibly diverted to nearby shivering people via
Hot Water to District Heating (7), with all output steam/hot water then sent to
Surface Reservoir (1) to be fully re-condensed by other heat exchangers, circled by
Crystalline Bedrock (10) keeping plant's water from wandering away, plus
Observation Well (9) which Wikipedia neglects to explain but I'd guess monitors
temperature of Porous Sediments, facilitating fine-tuning of plant operation

*But returning to **Geothermal 101's** definition of EGS:*

It's key phrase stated that the "E" of engineering required for an EGS plant involved:

"fracturing, pumping water into and out of the hot rock"

That sounds an awful lot like **natural gas fracking**

Suggesting that EGS reservoir creation might entail the same:

Massive potential for ground water / aquifer pollution

And for local air pollution

And for activation of dormant earthquake faults

Drawing from my note set on **Fossil Fuels** ([pptx](#) / [pdf](#) / [key](#))

and its detailed discussion of natural gas fracking techniques:

To fracture rock, frackers inject extremely high pressure water

It blows open interconnected cracks in the rock,

which then allows pockets of natural gas to drain out,

at least if cracks are not then clogged by hydrocarbon residues (e.g., tars)

and if they don't just collapse when the water pressure is reduced

To inhibit re-closure, fine sand is added to the water, tending to wedge cracks open ¹

To inhibit hydrocarbon clogging, solvents & detergents are added to the fracking water

(with their composition now not even disclosed to the U.S. public or government)

EGS field creation in low hydrocarbon rock might not require **those** chemicals

Finally, to inhibit **tight re-closure of cracks, their walls are also generally roughened**

via the addition of rock-etching acids to the fracking water

These chemicals probably **would** be desirable in creation of an EGS fields

1) The sand is called a "proppant" (so named because it "props" the cracks open?)

To explore this (and other) issues I found a massive MIT led study:

The Future of Geothermal Energy ¹

Echoing information above, that report's synopsis states (p. 1-1):

"Although conventional hydrothermal resources are used effectively for both electric and nonelectric applications in the United States, they are somewhat limited in their location and ultimate potential for supplying electricity.

Beyond these conventional resources are EGS resources with enormous potential for primary energy recovery **using heat mining technology**, which is designed to extract and utilize the earth's stored thermal energy."

That report moves on to then make a striking claim (p. 1-3):

"With a reasonable investment in R&D, EGS could provide 100 GW_e or more of cost-competitive generating capacity in the next 50 years."

A "cost competitive" 100 GW would equal 20% of today's U.S. power generation

1) https://www1.eere.energy.gov/geothermal/pdfs/future_geo_energy.pdf

The report additionally claims that:

(p. 1-4): "Field studies conducted worldwide for more than 30 years have shown that EGS is technically feasible in terms of producing net thermal energy by circulating water through stimulated regions of rock at depths ranging from 3 to 5 km. We can now stimulate large rock volumes (more than 2 cubic km), drill into these stimulated regions to establish connected reservoirs, generate connectivity in a controlled way if needed, circulate fluid without large pressure losses at near commercial rates, and generate power using the thermal energy produced at the surface from the created EGS system. **Initial concerns regarding five key issues - flow short circuiting, a need for high injection pressures, water losses, geochemical impacts, and induced seismicity - appear to be either fully resolved or manageable with proper monitoring and operational changes.**"

But it later concedes that:

(p. 1-21): "Extensive drilling for petroleum, geothermal, and mineral resources during the past century has demonstrated that **the largest heat resource in the Earth's crust, by far, is contained in rocks of low natural permeability.** Recovery of heat from such rocks at commercial rates and competitive costs is the object of the EGS program."

And seemingly undercuts the earlier claim that issues have been largely resolved:

(p. 1-21): As expected in the early development of any new technology, many lessons have been learned from 30 years of EGS field research in the eight countries listed above. For example, the initial concept of producing discrete hydraulic fractures has largely been replaced by stimulating the natural fracture system. **Although the goal of operating a commercialized EGS reservoir has not been achieved yet,** field testing has successfully demonstrated that reservoirs of sufficient size with nearly sufficient connectivity to produce fluids at commercial rates can be established.

*But what about EGS's need for something resembling **fracking**?*

Discussion of hydraulic fracturing is woven throughout the 372 page MIT EGS report

Addition of chemicals to fracturing fluids is also repeatedly mentioned

E.G., on page 4-45: "dilute acids" or on page 5-4: "other chemicals"

But in this otherwise very detailed report, in its chapter on Environmental Impacts,

the entire discussion of water pollution (8.2.2) spans **two short paragraphs**,

and it includes no specific reference to fracking chemicals employed

That left me VERY uneasy about possible parallels between EGS and fracking

I nevertheless recognize a key difference between those technologies:

To extract depletable natural gas, **new** natural gas fields must be continuously fracked

But to extract effectively undepletable earth heat, once an EGS field is created

it could provide ~ endless power, and it might do so from a smallish field in an

isolated rock formation, from which water pollution could not easily spread

How plausible is a 20% EGS contribution to U.S. power ?

In the spirit of "We can figure this out" let's try a quick calculation:

How much Geothermal heat is flowing up to the earth's surface?

Wikipedia ¹ specifies this as **65 mW / m²** on land (vs. 110 at ocean bottom)

The U.S. Geological Survey ² reports a "normal" number equivalent to 63 mW / m²

"Sustainable Energy without the Hot Air" ³ gives a similar value of 50 mW / m²

What fraction of that heat energy might we be able to capture?

The **Carnot Cycle** ⁴ gives an idealized maximum power extraction efficiency of

$$\sim (T_{\text{high}} - T_{\text{low}}) / T_{\text{high}} \times 100$$

For geothermal heat engines, $T_{\text{low}} \sim$ earth surface temperature $\sim 300^{\circ}\text{K}$

Deep ground might be 200°C hotter (i.e., 500°K) giving theoretical limit of

$$\text{Max geothermal efficiency} \sim (500 - 300 / 500) \times 100 \sim 40\%$$

Combining: Max extractable Geothermal power $\sim 65 \text{ mW} / \text{m}^2 \times 40\% = 26 \text{ mW} / \text{m}^2$

1) from Wikipedia's Geothermal Gradient webpage: https://en.wikipedia.org/wiki/Geothermal_gradient

2) USGS: Geothermal Energy: <https://pubs.usgs.gov/circ/1965/0519/report.pdf>

3) Sustainable Energy - Without the Hot Air: <https://www.withouthotair.com/>

4) Discussed in my note set: Generic Power Plant and Grid ([pptx](#) / [pdf](#) / [key](#))

The total land area of the U.S. is 9,147,593 km²

Multiplying that number ¹ by the preceding slide's capturable Geothermal energy/m²:

$$\begin{aligned}\text{Max U.S. Geothermal Power} &= 9,147,593 \text{ km}^2 \times 26 \text{ mW} / \text{m}^2 \times (10^6 \text{ m}^2 / \text{km}^2) \\ &= 2.4 \times 10^{14} \text{ mW} = 2.4 \times 10^5 \text{ MW} = 0.24 \text{ TW}\end{aligned}$$

As noted earlier, average total U.S. power consumption is ~ 1/2 TW

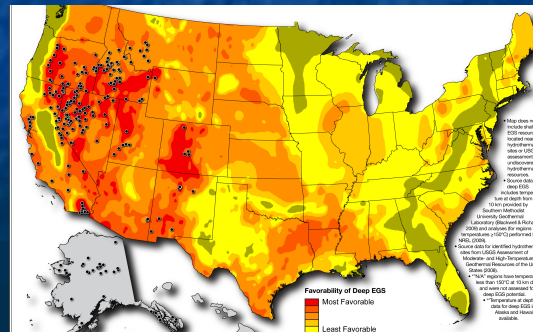
Thus, if 100% of U.S. land area were effectively used for EGS,

Geothermal could produce (at best) ~ .24 / 0.5 ~ 50% of U.S. power

OR, **to provide 20% of U.S. power** (as claimed in the report discussed above):

Geothermal would require ~ 2/5^{ths} of the total U.S. land (including Alaska)

Not likely! Especially when NREL rates HALF of the lower 48 **unfavorable** for EGS:



1) From Wikipedia: https://en.wikipedia.org/wiki/List_of_U.S._states_and_territories_by_area

Ocean Thermal Energy Conversion

Ocean Thermal Energy Conversion = OTEC

OTEC and Geothermal power have a lot in common - Both employ:

A turbine-electrical-generator driven by high pressure vapor
that was boiled by heat acquired from a **HOT** reservoir, and then
re-condensed by dissipating heat into a **COLD** reservoir

But OTEC's reservoirs are flipped:

The upper **HOT** reservoir is warm water near the **surfaces** of some oceans

The lower **COLD** reservoir is near-freezing water in the **depths** of some oceans

And because "HOT" ocean surfaces are not hot enough to boil water,

some other "working fluid" must be vaporized & condensed,

something that boils **between** deep & shallow seawater temperatures

The working fluid chosen in most OTEC projects is ammonia

<https://www.makai.com/ocean-thermal-energy-conversion/>

But Ammonia's one atmosphere boiling point is -33.34 °C

Which is below saltwater's **freezing** point

Meaning that even deep cold (liquid) seawater cannot condense ammonia

OTEC gets around this by pressurizing the ammonia, which elevates its boiling point

Developers seek ocean locations where warm (shallow) water and cold (deep) water

differ in temperature by at least 20 °C ^{1, 2}

And, to economically access that cold water, they want locations where

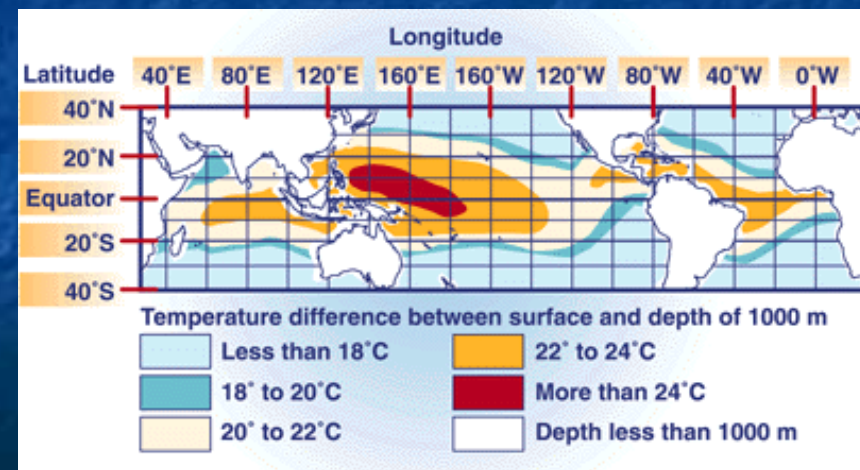
sufficiently cold water is no deeper than ~ 1000m

Satisfying BOTH requirements is difficult:

These conditions are met only within

the **Red / Orange** bands of

our **tropical oceans**: ¹



1) https://en.wikipedia.org/wiki/File:Temperaturunterschiede_Ozeane.png

2) <https://www.britannica.com/technology/ocean-thermal-energy-conversion>

And even 20 °C is not much of a temperature difference:

Thermodynamics says that thermal power conversion efficiency cannot exceed:

$$\sim (T_{\text{high}} - T_{\text{low}}) / T_{\text{high}} \times 100 \text{ (the highly idealized \& optimistic Carnot maximum)}$$

For **Geothermal Power** that efficiency was $\sim (500 \text{ °K} - 300 \text{ °K}) / 500 \text{ °K} \Rightarrow$ **40%**

Compared to OTEC where T_{low} is the temperature of deep **liquid** salt water,
which even with its salt, can't be much colder than $0 \text{ °C} = 273 \text{ °K}$

Thus, for ocean regions with the preceding map's $\Delta T = T_{\text{high}} - T_{\text{low}} = 20 \text{ °C}$,
the sea surface "high" temperature must be $\sim 20 \text{ °C} = 293 \text{ °K}$

Leading to a maximum **OTEC** thermal power conversion efficiency of:

$$\sim (T_{\text{high}} - T_{\text{low}}) / T_{\text{high}} \times 100 = (293 \text{ °C} - 273 \text{ °C}) / 293 \text{ °C} =$$
 7%

Thus, to **MATCH** the power output of Geothermal, OTEC plants must process

\sim six times more water, requiring some exceptionally large pipes & pumps!

But tropical islands DO often lack acceptable power alternatives

Small / isolated islands seldom have large local fossil fuel reserves,
or the rivers necessary for hydroelectric power,
or the large tracks land required for solar farms

Many such islands also lack the vulcanism supporting shallow Geothermal power,
or have cultures hesitant about exploiting vulcanism (e.g., as is possible in Hawaii)

Finally, dependence on tourism may make scenery-marring technologies unacceptable,
including any form of solar power, as well as both onshore & offshore wind power

The island potential of OTEC's was thus realized as early as the 1880's,
as evident in French Physicist Jacques Arsene d'Arsonval's Cuban test plants ¹

And in the latter half of the 20th century, test plants were built in locations ranging from
West Africa's Ivory Coast, to Japan's Nauru Island, to India's Tamil Nadu ^{1, 2}

This progression now continues in an Hawaiian test project:

1) https://en.wikipedia.org/wiki/File:Temperaturunterschiede_Ozeane.png

2) <https://www.britannica.com/technology/ocean-thermal-energy-conversion>

Kona Hawaii's ongoing OTEC test project:

OFFSHORE: Hot (18m deep) & Cold (1000m deep) seawater intakes / returns

ONSHORE: Heat exchangers boiling then re-condensing ammonia working fluid, its steam driving 24/7 turbine generators (100 KW now => 100 MW planned)

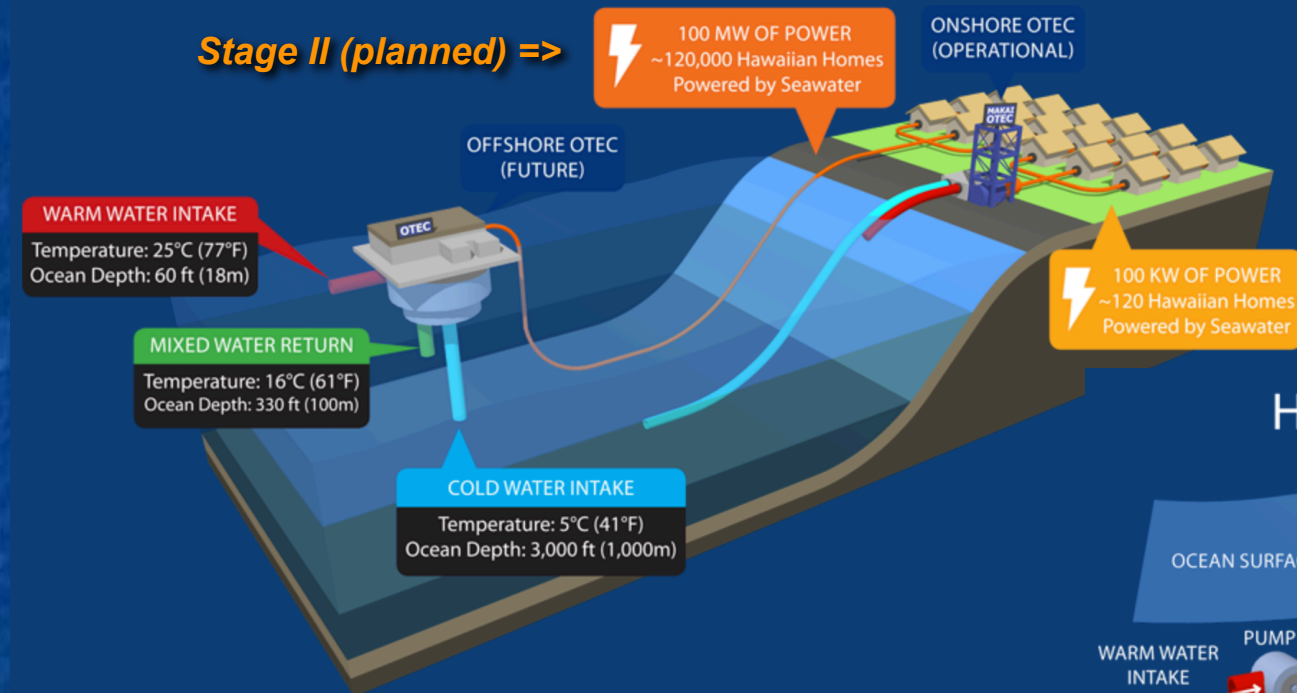
Stage II (planned) =>

100 MW OF POWER
~120,000 Hawaiian Homes
Powered by Seawater

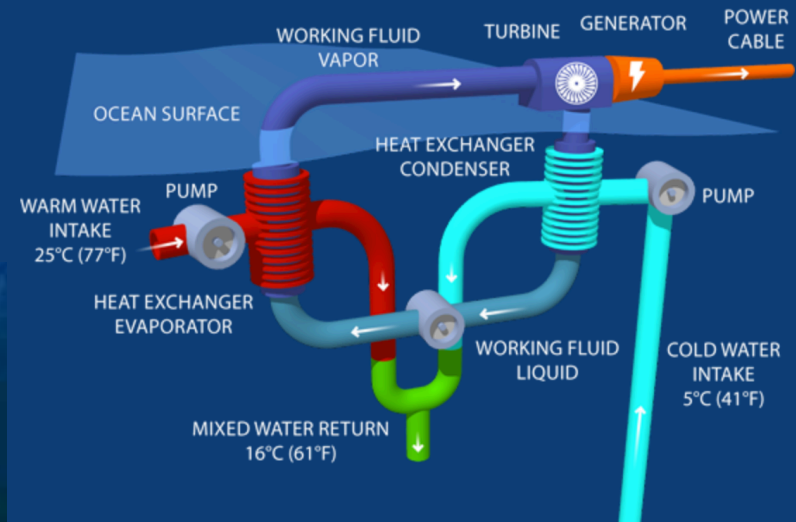
ONSHORE OTEC
(OPERATIONAL)

100 KW OF POWER
~120 Hawaiian Homes
Powered by Seawater

<= Stage I (2015)



HOW OTEC WORKS



A cooperative project involving Makai Corp ¹ & U. Hawaii's Natural Energy Research Institute ²

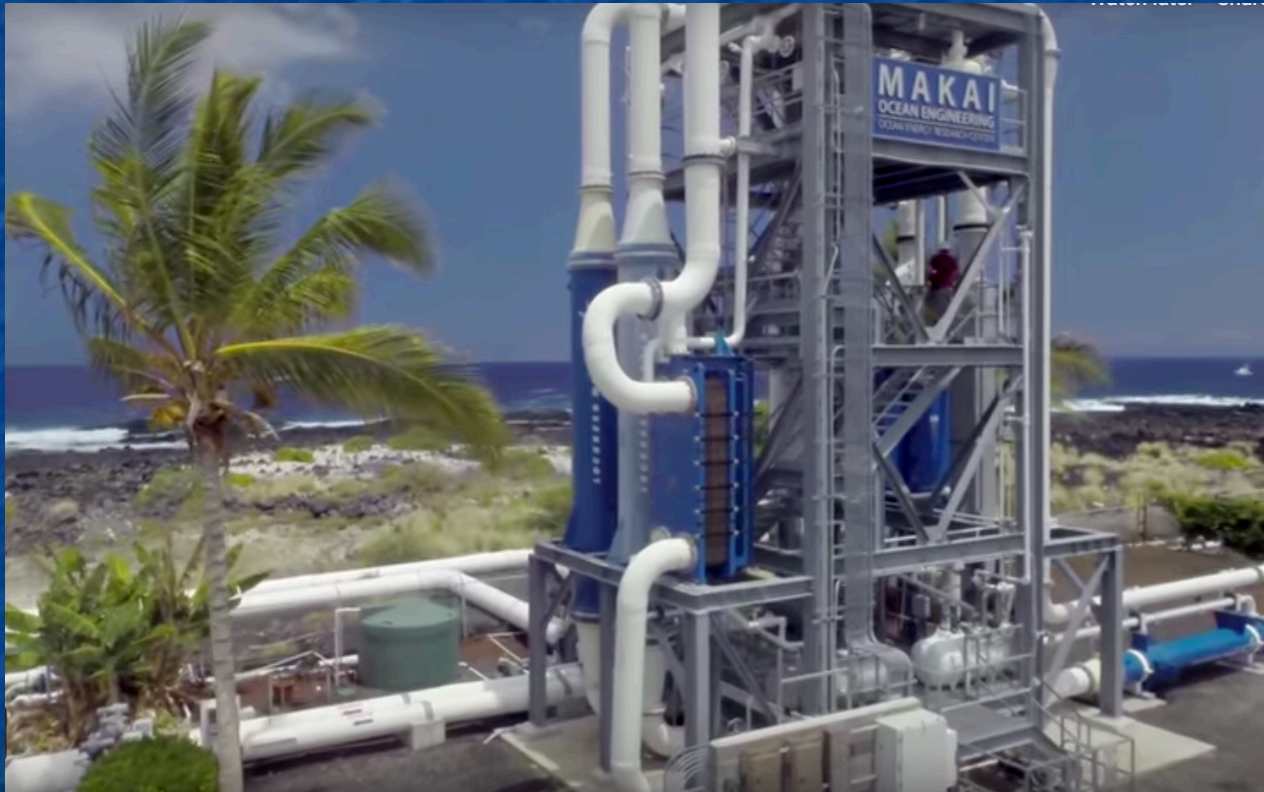
Figs. & Ref 1) <https://www.makai.com/ocean-thermal-energy-conversion/>

2) <https://www.hnei.hawaii.edu/>

A Makai video further explaining that test project:

Makai webpage embedding video: <https://www.makai.com/ocean-thermal-energy-conversion/>

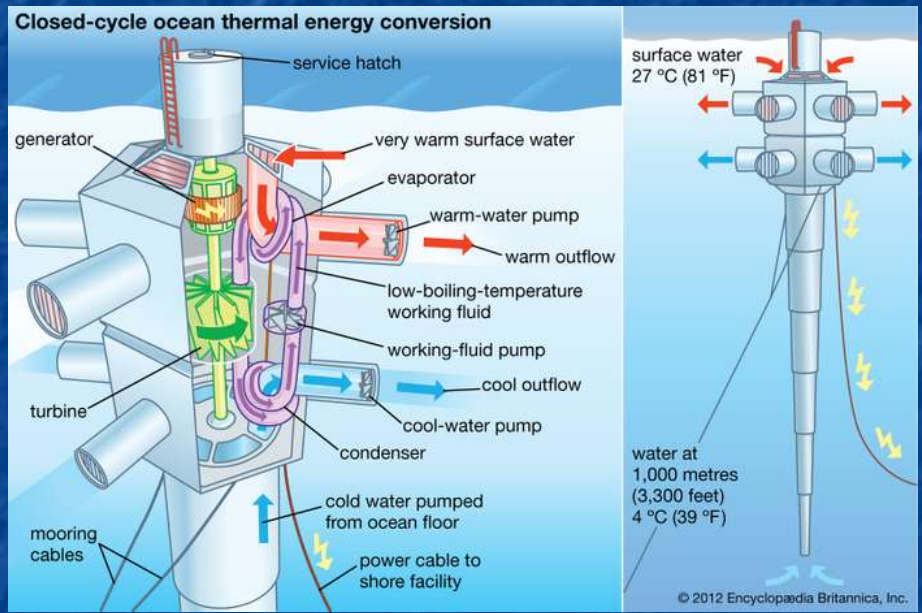
Direct YouTube link: <https://www.youtube.com/watch?v=LJV4d4XtHuo>



On & Offshore components **could** be combined within a floating tower

Which would eliminate long (expensive) runs of large-bore seawater pipe connecting onshore heat exchangers / turbine generators with

towers floating ~ 1000m above the necessary cold (but deep) seawater



*Encyclopedia Britannica:
Ocean Thermal Energy Conversion
Technology*
[https://www.britannica.com/
technology/ocean-thermal-energy-
conversion](https://www.britannica.com/technology/ocean-thermal-energy-conversion)

- Likely downsides? Higher cost of much more complex floating tower
- Higher cost of maintaining offshore heat exchangers & generators
- Possibility of offshore (in sea) leaks of ammonia working fluid

OTEC plants are now being considered in:

The Bahamas, Hainan China, Japan, U.S. Virgin Islands,

Kiribati, Korea, Martinique and The Maldives ¹

But the technology has yet to be extensively tested

Indeed, not a single commercial scale (100's of MW) plant has yet been built
(much less operated for an extended amount of time)

So the technology must still be considered to be in its infancy

Bottom lines regarding Ocean Thermal Energy Conversion?

Based on our admittedly very limited worldwide experience with OTEC, it appears that:

While OTEC may contribute strongly to power on isolated islands or mainland coasts,
impact within heavily populated continental areas seems unlikely

The Physics of Tapping into Water Power

*Water can provide power from either its **kinetic** or **potential energy**:*

Water's potential energy is captured by forcing **falling** water to do work

From high school physics, **gravitational potential energy** is:

$$E_{\text{gravity}} = M g h \quad \text{where } g = \text{Earth's surface gravity} \quad \text{and } h = \text{the water's height}$$

The **change in energy** after falling a distance Δh is then: $\Delta E_{\text{gravity}} = M g \Delta h$

And the **power** extractable from that falling water is:

$$\text{Power}_{\text{water_gravity}} = \Delta E_{\text{gravity}} / \Delta \text{Time} = M g \Delta h / \Delta t = (\text{mass of water falling} / \text{time}) g \Delta h$$

But the mass of water falling per time = (volume of water flow) x (density of that water)

Labeling that flow (a volume per time) as: F and the water's density as: ρ_{water}

We get for the power extractable from a falling flow of water:

$$\text{Power}_{\text{water_gravity}} = g \Delta h F \rho_{\text{water}} \quad \text{OR:} \quad \text{Power}_{\text{water_gravity}} = g \rho_{\text{water}} \text{Flow} \times \Delta h$$

Substituting in $g = 9.8 \text{ m} / \text{sec}^2$, $\rho_{\text{water}} = 1 \text{ g} / \text{cc}$, and using fact that $1 \text{ kJ} = 1 \text{ kW-sec}$:

$$\text{Power}_{\text{water_gravity}} = 9.8 \text{ (kW-sec} / \text{m}^4) \times \text{Flow} \times \Delta h$$

*To extract water's **kinetic energy**, you must force it to slow down*

For instance, by forcing that flowing water to turn a propellor connected to a generator

Again from high school physics, kinetic energy (K.E.) = (1/2) M x velocity²

making the **kinetic energy per volume of flowing water**:

$$(1/2) \rho_{\text{water}} \text{velocity}_{\text{water}}^2 \quad (\text{because water } M / \text{volume} = \text{density} = \rho_{\text{water}})$$

The water **power** passing through a plane (where we'll put the propellor) is then:

$$\text{Power}_{\text{water flow}} = (\text{Water K.E.} / \text{Volume}) \times (\text{Volume of water through that plane} / \text{time})$$

However: Volume passing plane / time = (Perpendicular area of plane) (velocity_{water})

Combining all three expressions yields a possibly surprising result:

$$\text{Power}_{\text{water flow}} = (1/2) \rho_{\text{water}} \text{velocity}_{\text{water}}^2 \times \text{Area}_{\text{plane}} \times \text{velocity}_{\text{water}} \quad \text{or rearranging:}$$

$$\text{Power}_{\text{water flow}} = (1/2) \rho_{\text{water}} \times \text{Area}_{\text{plane}} \times \text{velocity}_{\text{water}}^3$$

Yes: Power of a flow crossing a plane varies as flow's velocity cubed (not squared)!

Because, while a particular volume of flow has kinetic energy proportional to v^2 , the flow is delivering those volumes to that plane at a rate also proportional to v

*But you can't hijack **all** of a flow's kinetic energy - at least not continuously*

You could do it momentarily by suddenly inserting a barrier into its path

Which would give that barrier a kick convertible into electrical power

But, once stopped by the barrier, no more flow would arrive => No more power out!

What you **could do** is instead insert a propellor geared to a generator such that

the water manages to flow through it at say, **half its original velocity**

Note here that because water is almost incompressible, to maintain this situation

part of the incoming flow would have to divert out and around that propellor

Nevertheless, you'd have slowed part of the flow to half its velocity, resulting in:

$$\text{Lost power} = (1/2) \rho_{\text{water}} \times A_{\text{plane}} \times v_{\text{water}}^3 - (1/2) \rho_{\text{water}} \times A_{\text{plane}} \times (v_{\text{water}} / 2)^3$$

$$= (1/2) (1 - 1/8) \rho_{\text{water}} \times A_{\text{plane}} \times v_{\text{water}}^3$$

$$= (7/8) \text{ Water's incoming flow power} = \text{Power to your generator!}$$

*(At least for the **fraction** of the flow that continued onward through your propellor)*

The value & utility of those water power formulas

These formulas evaluate the **power input** to water power plants

Which determines an **absolute upper limit on power output** from such plants,

This can often expose exaggerated claims made for new energy technologies

But their value in estimating an **actual power plant's output** is mixed

The difficulty is in evaluating the conversion efficiency of a particular type of plant

Gravitational energy is a best case, because its conversion to electrical energy is **uncomplicated**

Real hydroelectric plants thus convert 80-90% of input power calculated by the 1st formula

Flow energy is more of a worst case, because its conversion is **so very complicated**

As noted above, to keep from killing the flow, you can only partially slow it, meaning that:

- 1) You can capture only a fraction of its total incoming kinetic energy (2nd formula)
- 2) And much of the flow's material must bypass your converter (e.g., the turbine)

The latter point implies that turbines will have to be widely spaced,

which further reduces the fraction of flow's total input energy that can be captured

These distinctions mean that, in this note set:

For Tidal Barrages converting water's gravitational potential energy,

I will use the relevant formula to optimize a Tidal Barrage's design

I will further calculate the likely total power produced by a real life Tidal Barrage

Whereas for Tidal Flow farms converting water's kinetic energy,

I will use the relevant formula to identify flows maximizing input power

But I cannot accurately estimate total power produced by a specific turbine farm

THAT instead requires extremely complex computer-based modeling

I will discuss one such study which calculates maximum likely output power

from turbine farms planned for a specific location,

but which provides frustratingly incomplete information about the

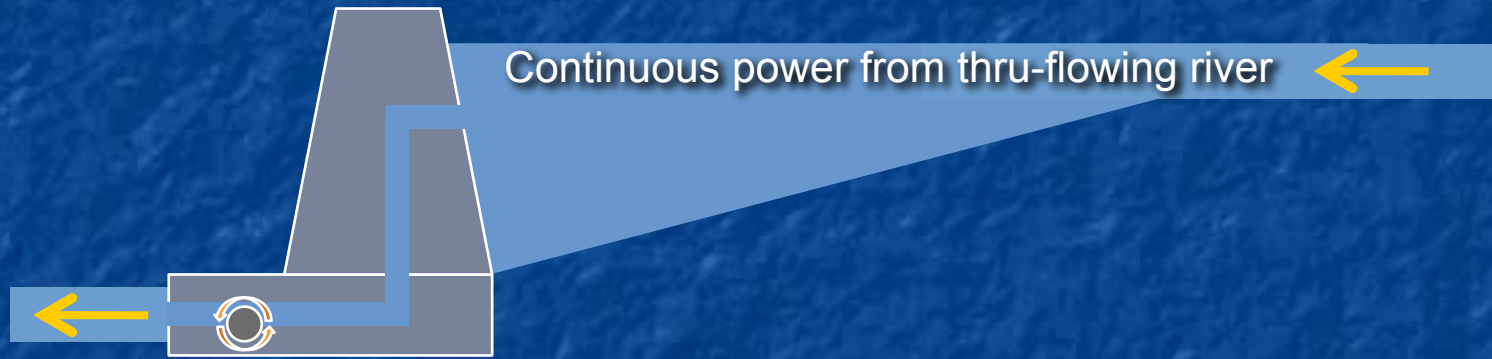
turbine & farm designs yielding that maximum power output

Tidal Barrage Power

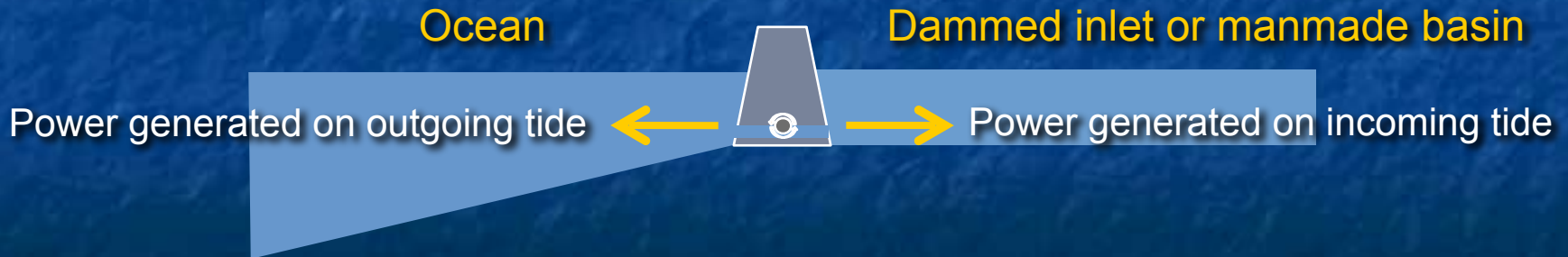
Covered more completely in my note set on Hydro Power ([pptx](#) / [pdf](#) / [key](#))

A Tidal Barrage is a dam NOT filled by an incoming river

Conventional Dam: Outgoing River / Large Dam / Large Reservoir / Incoming River



Tidal Barrage: Big Tidal Ocean / Small Dam / Small Reservoir



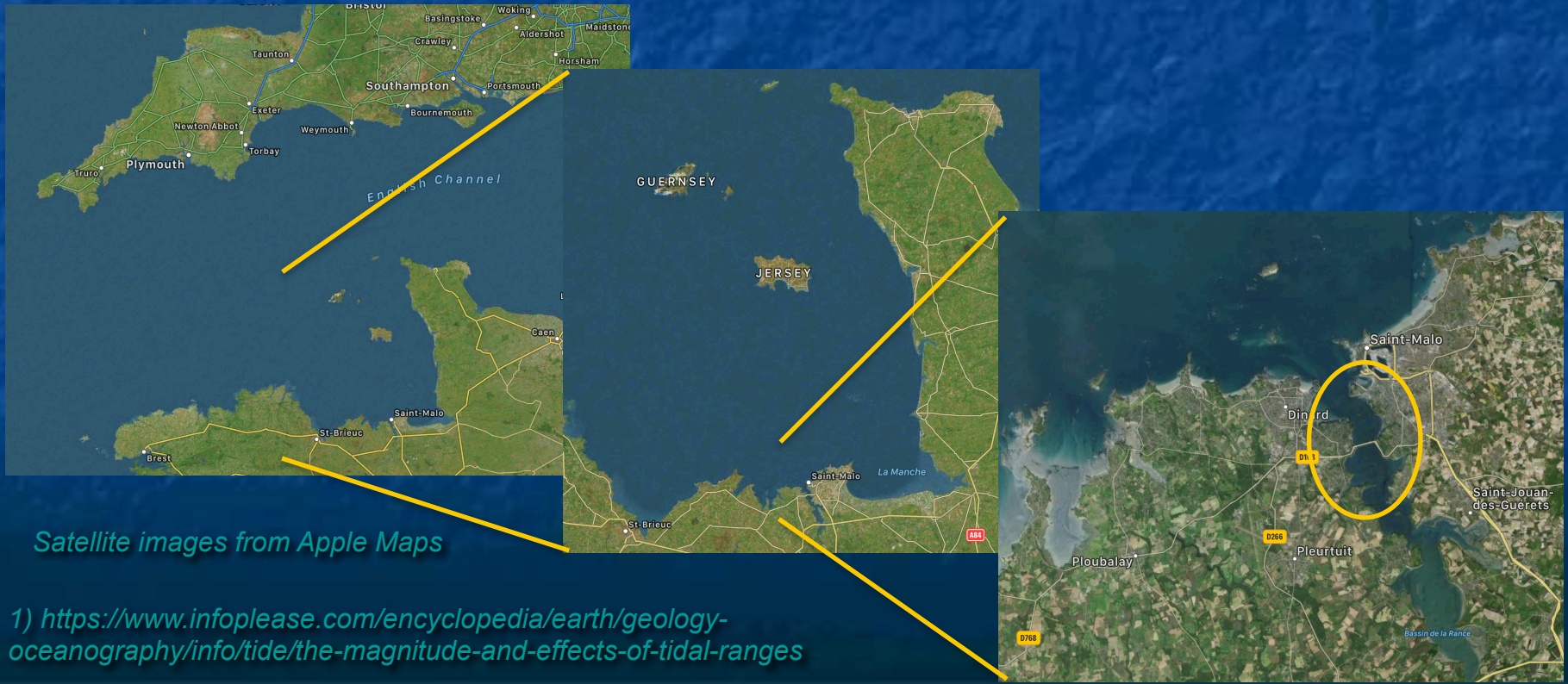
Higher the tides - Higher a Tidal Barrage's power

But as they reach our coasts, tides typically rise and fall by only **2-3 meters** ¹

The solution: Seek out nature's many **atypical** coastal tides

such as those created where tides are squeezed into narrowing inlets & bays

Such funneling occurs on several scales as tides approach France's Brittany coast, leading to **8 meter** tides in the Rance River estuary, adjacent to Saint Malo



1) <https://www.infoplease.com/encyclopedia/earth/geology-oceanography/info/tide/the-magnitude-and-effects-of-tidal-ranges>

The (circled) road crossing Saint Malo's estuary was in fact a Tidal Barrage:

Built in 1966, it remained the **world's largest tidal barrage** until 2011

But as seen below, it appears to be little more than a small / low river crossing

As with solar & wind, Tidal Power output mirrors nature's cycles of power input

But with ~ two tides per day, and peak flows for both incoming & outgoing tides,

Tidal Barrage power plants produce **four daily cycles** of power

At the peaks in those tidal flows, the **Rance Tidal Barrage** produces **240 MW**



But its cycle-averaged power is **57 MW**

Which is small for a power plant

But not bad for a plant with such a

small footprint & visual impact

Which exploits essentially free "fuel"

And produces no exhaust or pollution

But why is a small 1966 plant still the prime example of this technology?

Especially when "small footprint" suggests low-cost plant construction & maintenance

Which, combined with "free fuel" and "no exhaust or pollution,"

sounds like a recipe for near certain power plant success

The disconnect may lie in the Rance Barrage's "small footprint" which required construction of only a short low dam across a small natural estuary

Large Tidal Barrages would instead require long dams across large estuaries

But we already use large natural estuaries for LOT of things, including:

shipping and/or fishing and/or as ecological and/or recreational reserves

These would be disrupted or totally blocked by conversion into Tidal Barrages

And it is likely that artificial single-purpose Barrages would be hugely expensive

The scale of those challenges is suggested in these figures:

Which depict **intensely controversial Tidal Barrages** now proposed in Britain

Proposed Tidal Barrages obstructing the heavily trafficked Severn River estuary:

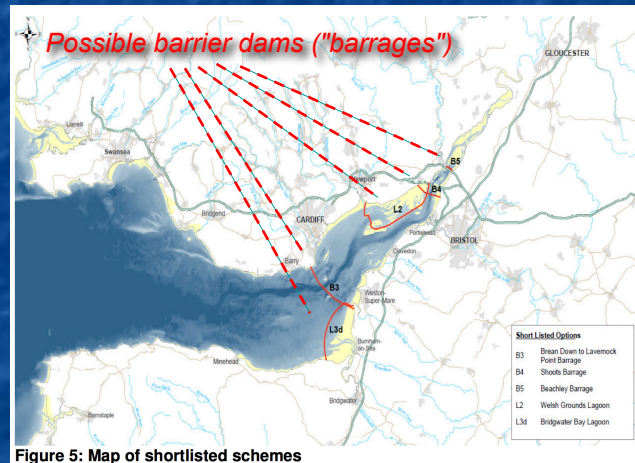
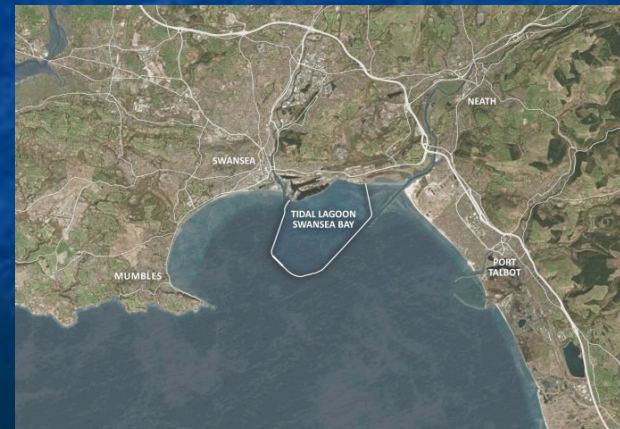


Figure 5: Map of shortlisted schemes

https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/50064/1_Feasibility_Study_Conclusions_and_Summary_Report_-_15_Oct.pdf

Would-be drone & satellite views of a huge looping tidal barrier off the Swansea coast:



<https://www.newscientist.com/article/2117792-uk-urged-to-push-ahead-with-world-first-tidal-lagoon-power-plant/>

<http://www.tidallagoonpower.com/projects/swansea-bay/>

UK feasibility studies are discussed in my Hydropower Power note set ¹

Highlights of those often strongly disagreeing UK studies include:

Estimates that power from those mega Tidal Barrages might

cost from two to seven times more than we now typically pay in the U.S.

i.e., from 0.17 to 0.6 £ / kW-h (equivalent to ~ 22 - 78 ¢ / kW-h)

And one UK government report claims that:

"A tidal power scheme . . . is high cost and high risk in comparison to other ways of generating low-carbon electricity"

"In most cases other renewables (e.g. wind) and nuclear power represent better value"

"Scale and impact of a scheme would be unprecedented in an environmentally designated area"

"Fish are likely to be severely affected with local extinctions and population collapses predicted"

But one must recognize that (as in the U.S.) UK green energy has been intensely politicized

With seemingly every negative report countered by a positive report,

in a battle that (to my personal knowledge) has raged for at least two decades,

A simpler question: What is the upper limit on Tidal Barrage power?

The Barrage is powered by water flowing through its turbines

But the ultimate source of that power is the gravitational potential energy added to the water lifted into the Barrage by the incoming tide

That lifted water is upper right in this figure - which depicts a tide of height H :

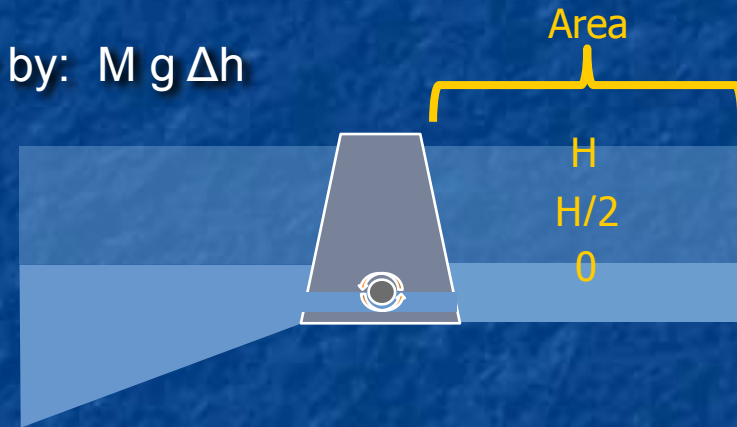
The added water has mass: $M = \rho_{\text{seawater}} \times \text{Area} \times H$ ($\rho_{\text{seawater}} = 1029 \text{ kg / m}^3$) ¹

Lifting increased each unit of water's potential energy by: $M g \Delta h$

But while water at the top rose by H

Water at the bottom base rose by 0

Making for an average rise of $\Delta h = H/2$



The total potential energy added within the Barrage was thus:

Added Tidal Energy = $M g \Delta h = (\rho_{\text{seawater}} \times \text{Area} \times H) g (H/2)$

Rearranging: **Added Tidal Energy = $\rho_{\text{seawater}} g \text{Area } H^2 / 2$**

1) <https://en.wikipedia.org/wiki/Seawater>

That much energy would be capturable over each tidal cycle

The average available tidal-driven power would thus be:

$$\text{Power}_{\text{tidal}} = (\text{Tidal Energy}) / (\text{Tidal Cycle Time})$$

For a typical tidal cycle time of ~ 12 hours = 43,200 seconds:

$$\begin{aligned}\text{Power}_{\text{tidal}} &= [\rho_{\text{seawater}} g \text{ Area } H^2 / 2] / [\text{Tidal Cycle Time}] \\ &= [(1029 \text{ kg} / \text{m}^3) (9.8 \text{ m} / \text{s}^2) \text{ Area } H^2 / 2] / [43,200 \text{ s}] \\ &= 0.12 (\text{kg} / \text{m}^2 - \text{s}^3) \text{ Area } H^2\end{aligned}$$

Using the definition of a Watt: $W = J / s = (\text{kg m}^2 / \text{s}^2) / s = \text{kg m}^2 / \text{s}^3$ we then get:

$$\text{Power}_{\text{tidal}} = 0.12 \text{ Watts} / \text{m}^4 \times (\text{Area} \times H^2)$$

For a 1 km² Tidal Barrage with a worldwide average coastal tide ¹ of 2.5m:

$$\text{Power}_{\text{tidal}} = 0.12 \text{ Watts} / \text{m}^4 \times (10^6 \text{ m}^2 \times 6.25 \text{ m}^2) = 0.75 \text{ MW}$$

But the Rance Tidal Barrage has an area of 22.5 km² with 8m tides: ¹

So its time-averaged available tidal power should approach:

$$\text{Power}_{\text{tidal}} = 0.12 \text{ Watts / m}^4 \times (22.5 \times 10^6 \text{ m}^2 \times 64 \text{ m}^2) = \mathbf{172 \text{ MW}}$$

Given that the real Rance Tidal Barrage produces an average of **57 MW**,

this suggests that real-world Tidal Barrage power conversion efficiency is ~ 33%

That is identical to the real-world efficiency of coal power plants

But more closely related Pumped Storage Hydro plants achieve 80%

Suggesting shortcomings in my simple potential-energy-based model

Using the real-world Rance power, but adjusting for more typical 2.5m tides:

$$\text{Power}_{\text{tidal}} / \text{km}^2 = 57 \text{ MW} \times (2.5 \text{ m} / 8 \text{ m})^2 / 22.5 \text{ km}^2 = \mathbf{0.25 \text{ MW} / \text{km}^2}$$

which is **ten times larger** than Geothermal's $26 \text{ mW} / \text{m}^2 = \mathbf{0.026 \text{ MW} / \text{km}^2}$

But at least **fifty times smaller** than Solar's $12\text{-}50 \text{ W} / \text{m}^2 = \mathbf{12\text{-}50 \text{ MW} / \text{km}^2}$ (Ref. 2)

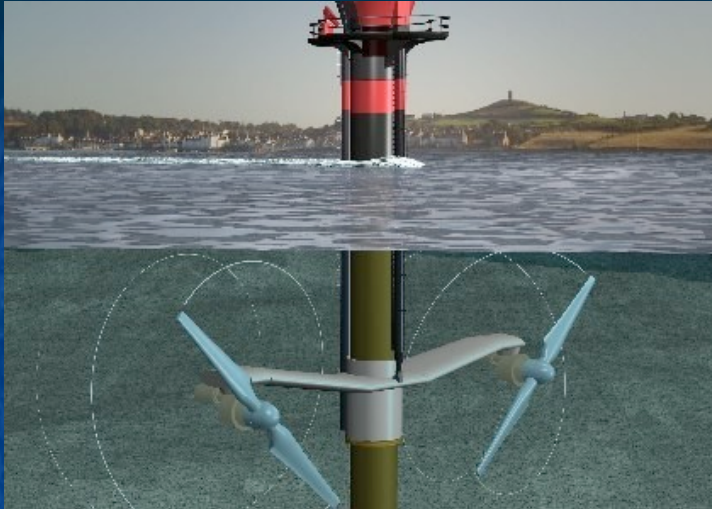
Bottom Line: Barrages will require HUGE AREAS for nation-scale power

1) http://en.wikipedia.org/wiki/Rance_Tidal_Power_Station

2) My note set: *Today's Photovoltaic Solar Cells* ([pptx](#) / [pdf](#) / [key](#))

Tidal Stream Power

Tidal Stream power taps into the kinetic energy of flowing water



<http://subseaworldnews.com/2012/01/17/uk-seagen-tidal-turbine-gets-all-clear-from-environmental-studies/>



<https://www.nature.com/news/power-from-the-oceans-blue-energy-1.15045>

This Tidal Stream turbine, in Northern Ireland's **Strangford Loch** produces **1.2 MW**

Which, disappointingly, is only **~ 1/500th of an typical U.S. Power Plant**

Or **1/8th to 1/6th the power of a single modern commercial wind turbine**

Tidal Stream turbines resemble wind turbines in their conversion of **kinetic energy**

Both types of flow transport kinetic energy at a rate yielding a power of:

Power_{flow} = (1/2) ρ x Area x velocity³ where ρ is the density of the flow

But water's density is ~ 1000X that of air:

So why don't water flows carry 1000X the power of wind flows?

Because while wind velocities easily reach several ten's of kilometers per hour,
tidal flows are typically only several kilometers per hour

In the formula, this gives wind flows a $(10)^3 = 1000X$ velocity cubed advantage

Which exactly offsets water flow's 1000X density advantage

Tidal Stream power plants must thus seek out exceptionally fast tidal flows

Where, given the flow power equation's factor of velocity cubed,

short extremely fast flows may deliver MORE power than **long fast** flows

Where might one find exceptionally fast, if not always sustained, tidal flows?

Where tides from a **large bay** flow through a **narrow passage** to the ocean,

My west coast roots call to mind two U.S. candidates:

- The mouth of Puget Sound (The Strait of Juan de Fuca) and
- The mouth of San Francisco Bay (the "Golden Gate")

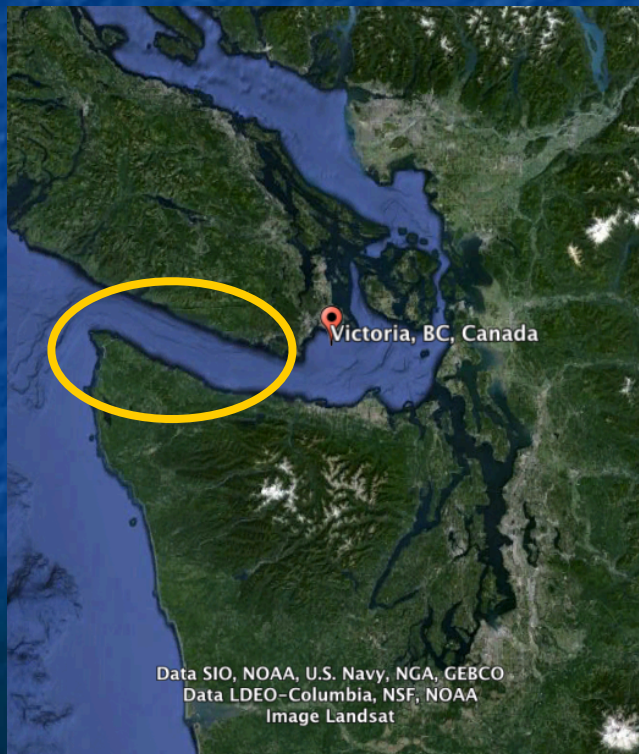
The Golden Gate has the larger bay to inlet size ratio:

Leading to tidal large whirlpools forming at the north side of the Golden Gate

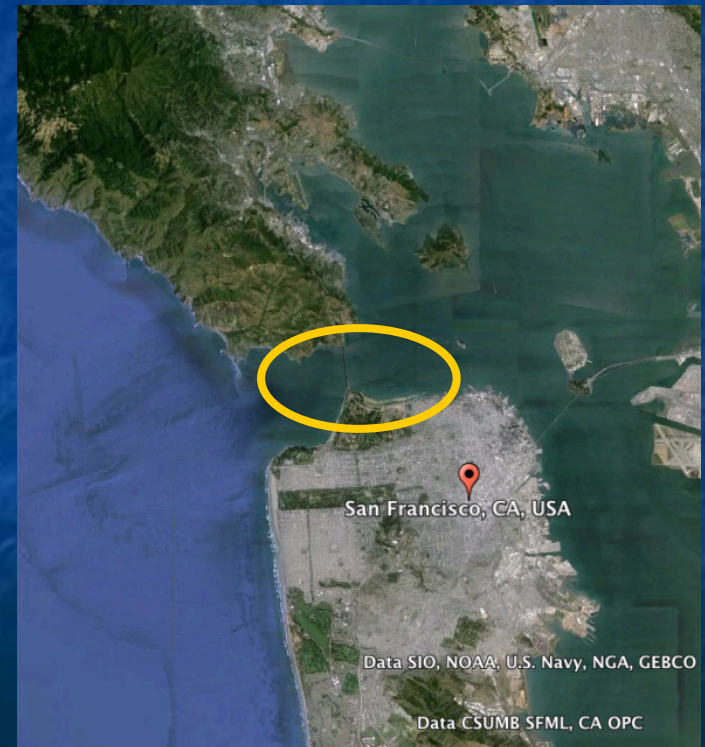
Which, as impressive as they are when viewed from the bridge above,

are little short of terrifying when viewed from the deck of a small sailboat

(from which I once had the "opportunity" to view them)



Google Earth



But some weird physics can provide for even faster flows

For instance, in Nova Scotia Canada's Bay of Fundy

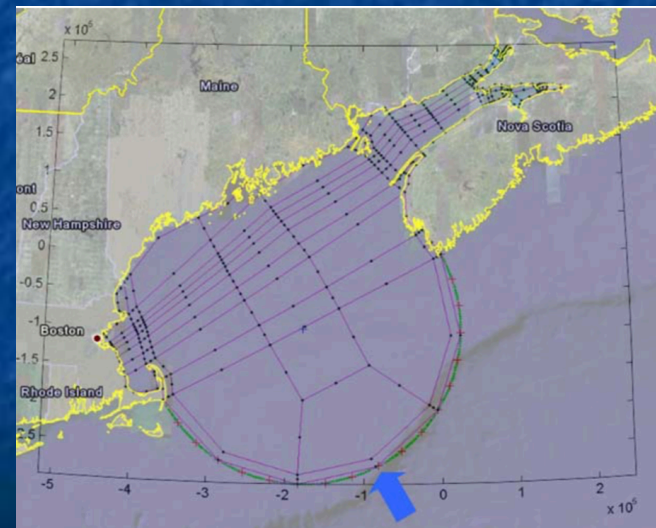
Fundy's 160 million tonne tidal flow exceeds the combined flow of all the world's rivers

That flow produces tides rising and falling up to **16 meters** (~ 58 feet),

and flowing at speeds up to **5.1 m / s = 18.4 km / hr (11.4 mph)** ¹

This is the product of a bathtub-like sloshing phenomenon called **Tidal Resonance**

The "bathtub" is formed by the Bay of Fundy + Gulf of Maine + Continental shelf ²



Ref 1) <https://www.oceannetworks.ca/observatories/atlantic/bay-fundy-minas-passage>

Ref 2 / Figures) https://www.researchgate.net/publication/49115803_HARBOR_RESONANCE_A_COMPARISON_OF_FIELD_MEASUREMENTS_TO_NUMERICAL_RESULTS

49115803_HARBOR_RESONANCE_A_COMPARISON_OF_FIELD_MEASUREMENTS_TO_NUMERICAL_RESULTS

Acting like an ultra-broad wave, a tide comes in off the deep ocean

That incoming tide raises the water level in the Gulf of Maine, some of which

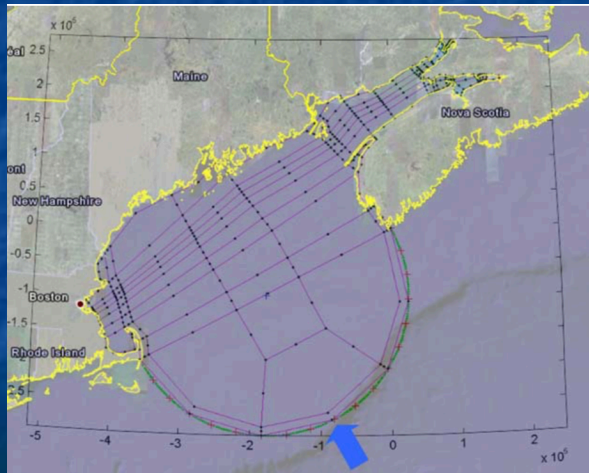
is then driven into the narrowing Bay of Fundy, at the upper right

As the tide retreats, water pours back out of the Bay as a broad wave traveling SW

Some of that wave eventually rebounds off Massachusetts and its Cape Cod

Other spreading parts reflect back at the edge of the Continental Shelf

These combined reflections then move back toward the coast & Bay of Fundy



But due to the Gulf + Bay geography, this

ECHO of the **original tide** returns ~ 12 hours later

and it thus **adds to the subsequent tide**

= **Tidal Resonance ~ Bathtub sloshing** ^{1, 2}

Ref 1 / Figure: https://www.researchgate.net/publication/49115803_HARBOR_RESONANCE_A_COMPARISON_OF_FIELD_MEASUREMENTS_TO_NUMERICAL_RESULTS

Ref 2) https://pdfs.semanticscholar.org/9c3b/d3750c1c3e02b331d1170ca7254eb57be302.pdf?_ga=2.61210730.738248831.1576318026-1968116301.1576318026

Such echo-induced resonances occur for all types of waves

Including light waves, sound waves, seismic waves, and even "electron waves"

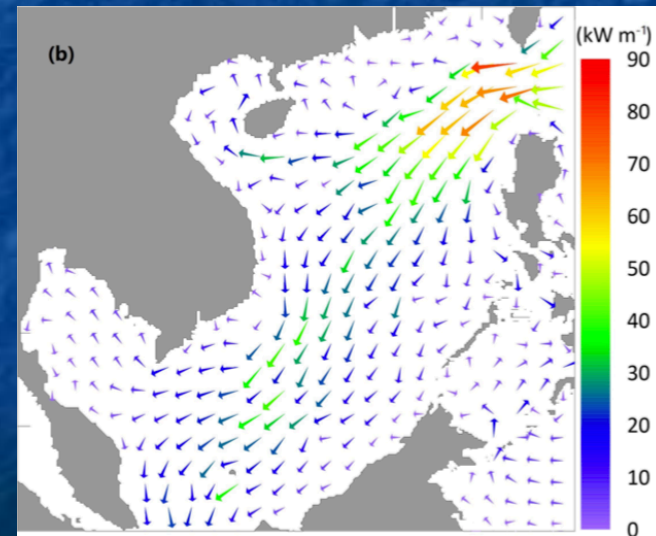
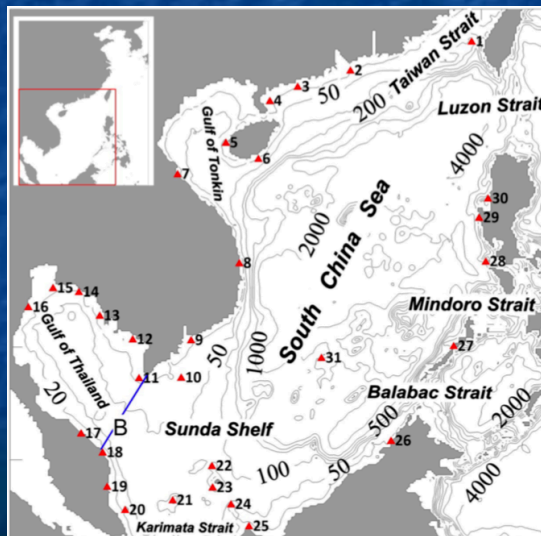
Light wave resonances enable the Internet's backbone of fiber optics

Seismic wave resonances can topple poorly conceived skyscrapers

Electron wave resonances are the very foundation of "Quantum Mechanics" ¹

Relevant to Tidal Stream power, varying degrees of Tidal Resonance occur worldwide

Including in the Gulf of Thailand (a.k.a. the "South China Sea"): ²



1) For more about wave resonance (including videos) see my Nanoscience note set [Electron Waves](#) ([pptx](#) / [pdf](#) / [key](#))

2) <https://www.ocean-sci.net/15/321/2019/>

But what sorts of turbines could survive in such super-tidal locations?

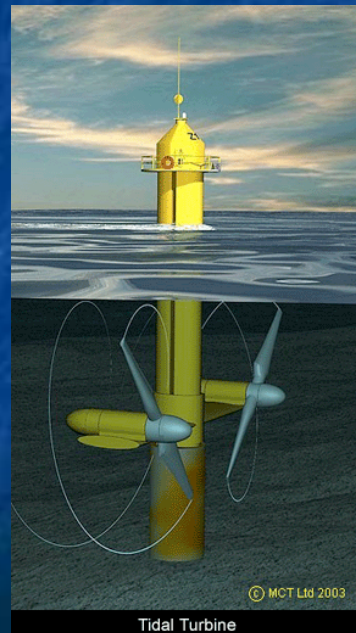
As noted earlier, even **normal** inlets & bay mouths are busy places, with all sorts of huge slowly moving (and thus only weakly steerable) ships passing through

Making it very likely that Tidal Stream turbines such as these designs (real & proposed), might have very short - if possibly very exciting - operational lifetimes

(That's if shipping & fishing interests allowed them to be **built** in the first place)



<http://www.bbc.co.uk/news/uk-wales-north-west-wales-11037069>



Tidal Turbine

<http://www.global-greenhouse-warming.com/tidal.html>



<http://www.fujitaresearch.com/reports/tidalpower.html>

Bottom-hugging designs are far more plausible:

They overcome the common trap of mimicking tall willowy wind turbines, substituting

compact & heavy turbines that sink and cling to the ocean bottom

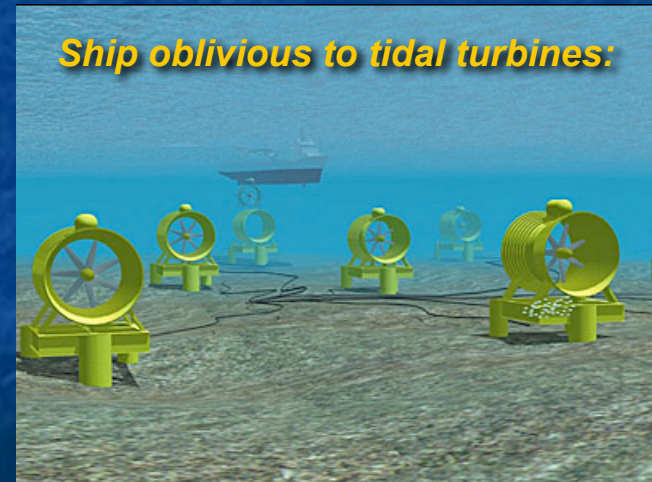
Eliminating any surface obstruction to navigation (= target for ship collisions) AND

Removing the need for VERY hard to construct **undersea foundations** (=\$\$\$)

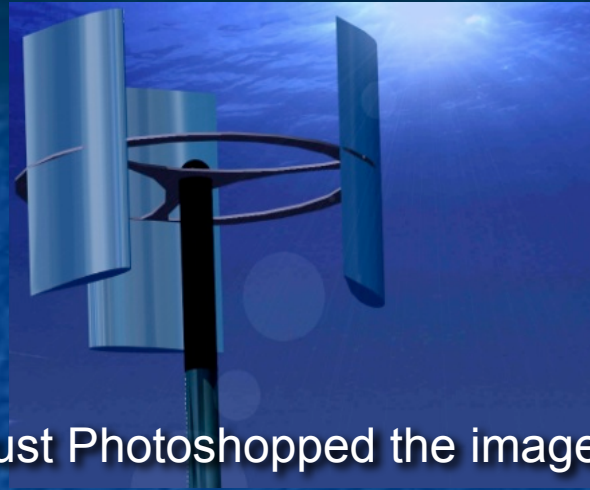
I.E., instead of this:



Something more like this:



Vs. less well thought out proposals:



http://www.esru.strath.ac.uk/EandE/Web_sites/10-11/Tidal/tidal.html

Here someone apparently just Photoshopped the image

of a **vertical axis wind turbine** onto an **undersea background**

The only advantage of such **wind turbines** is that wind from any direction turns them

But tides flow in **one** direction (and its opposite), so why use this flimsy design?

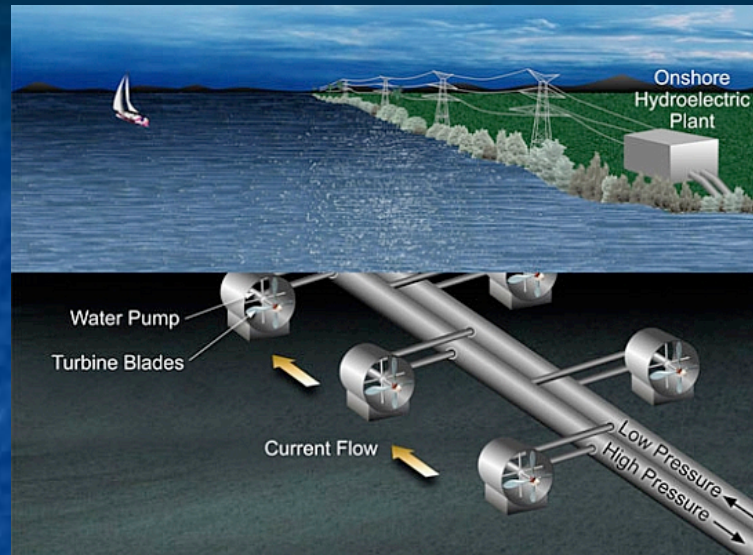
Further, the **force** on a turbine = Flow momentum transferred from fluid / time

$$= (\text{momentum/volume}) (\text{volume/time}) = (\rho v) (\text{Area } v) = A \rho v^2$$

$$\Rightarrow (1000X) (1/10X)^2 = 10X \text{ times the force on comparable wind turbine}$$

So a corresponding tidal turbine would have to be ten times thicker / stronger!

Vs. just plain dumb proposals:



<http://www.ecofriend.com/eco-tech-nasa-s-jpl-develops-a-cost-effective-way-to-harness-ocean-energy.html>

Instead of producing electricity AT the undersea turbines,
and then delivering it to shore via simple efficient ELECTRICAL CABLES

This proposal suggests using ocean bottom tidal turbines to pump water,
through BIG LONG PIPES,
all the way to an onshore hydro power station

Can you imagine the energy lost to friction & turbulence in those long pipes?

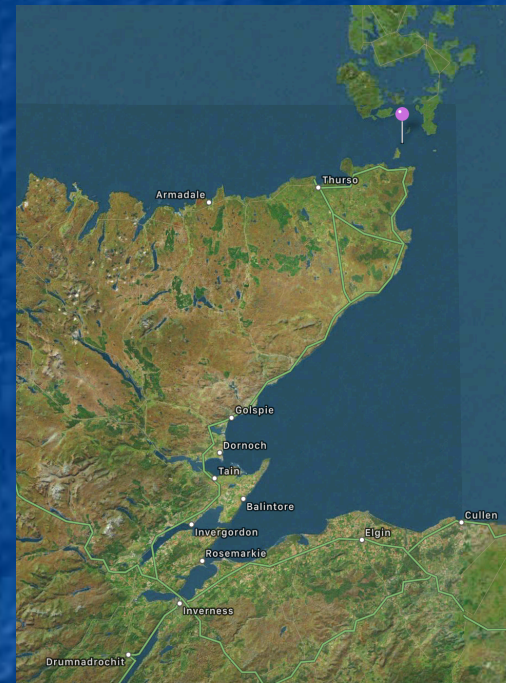
(to say nothing of the cost / difficulty in laying down those long pairs of pipes!)

Can we identify apparently successful / proven Tidal Stream designs?

MeyGen in Scotland's **Pentland Firth** is the largest ongoing Tidal Stream project

That "Firth" is the strait separating mainland Scotland from the Orkney Islands

Its up to 5 m/s water flow speeds equal those found in the Bay of Fundy ^{1, 2}



*Satellite Images
from Apple Maps*

The project's Phase IA placed four 16m diameter, 1.5 MW turbines on the ocean floor ³

The first turbine became fully operational in **2017**, and the fourth by **2018**

1) <https://en.wikipedia.org/wiki/MeyGen> 2) <https://www.oceannetworks.ca/observatories/atlantic/bay-fundy-minas-passage>

3) <https://www.theguardian.com/uk-news/2016/sep/12/worlds-first-large-scale-tidal-energy-farm-launches-scotland>

Its four operational turbines are indeed compact, foundation-less, bottom-clinging



Figure from reference 3

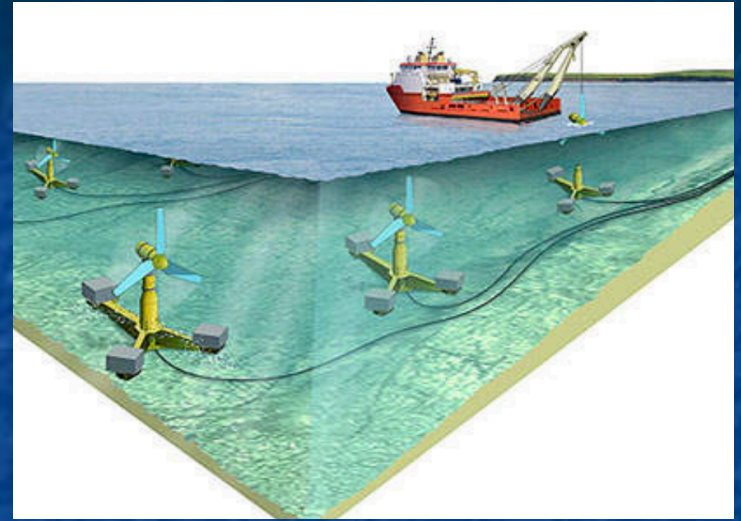


Figure from reference:4

Phase 1B (now in progress) will bring the project's total turbine count to **8** ^{1, 4}

Phase 1C (targeting 2021) would bring turbine count to **49** & capacity to **86 MW** ¹

A proposed 2nd phase would bring turbine count to **269** and capacity to **398 MW** ⁴

with the total installation then occupying **3.3 to 3.5 km² of ocean floor** ^{3, 4}

1) <https://en.wikipedia.org/wiki/MeyGen>

2) <https://www.theguardian.com/uk-news/2016/sep/12/worlds-first-large-scale-tidal-energy-farm-launches-scotland>

3) <https://www.powermag.com/meygen-array-sets-global-records-for-harnessing-tidal-power/?printmode=1>

4) <https://www.power-technology.com/projects/pentland-firth-tidal-power-plant-scotland/>

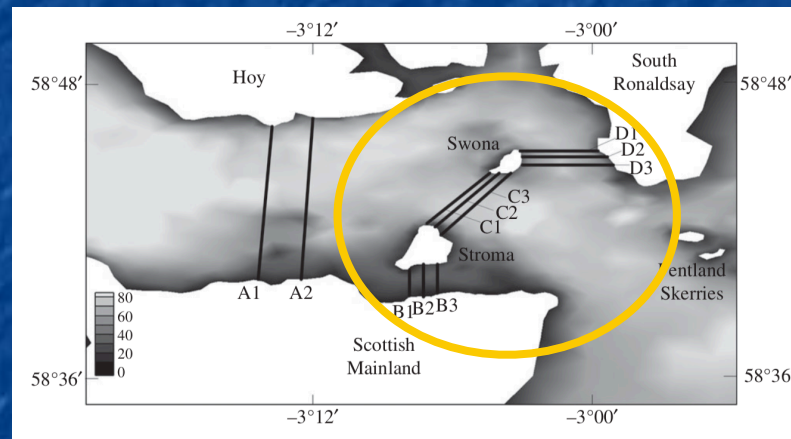
It's claimed Pentland Firth could provide 43% of Scotland's power ¹⁻³

This comes from a 2013 study involving U. Oxford & Edinburgh researchers ^{1, 2}

Which, based on modeling of Firth water flows in the presence of turbine farms,

concluded that **1.9 GW of Tidal Stream power** might be harvestable via:

"three rows of turbines extending across the entire width of the Pentland Firth and blocking a large fraction of the channel"



1) <https://royalsocietypublishing.org/doi/full/10.1098/rspa.2013.0072>

2) <https://www.eng.ed.ac.uk/about/news/20140128/tidal-energy-potential-pentland-firth-revealed>

3) <https://www.bbc.com/news/uk-scotland-north-east-orkney-shetland-25800448>

Turbine patterns & spacings were among the input parameters in this study

But I did not find final estimates of the number of turbines required for 1.9 GW

Extrapolation from MeyGen suggests: $269 \times (1.9 \text{ GW} / 398 \text{ MW}) \sim 1300$ turbines

But do these MeyGen turbines really qualify as "successful and proven?"

Only 4-8 turbines are now in operation, the oldest operating for only ~ three years

For Tidal Stream power to succeed, turbines had better operate for **dozens** of years, with little or no maintenance (likely requiring them to be hauled up off the seabed)

Further, as noted above, water turbines must withstand huge tidal forces

suggesting the use of the very best high high-strength steels,

which, unfortunately, start rusting immediately upon contact with salt water

causing joints, seals and bearings to freeze up, if not crumble apart

(to say nothing of the catastrophe of salt water + electronics) ¹

Earlier projects in the Bay of Fundy provide an apocryphal history:

1) See this story of the Jaws' robotic shark: <https://www.tested.com/art/movies/456576-robot-shark-technology-jaws/>

A Brief / Disappointing History of Tidal Stream Power in the Bay of Fundy:

It is mostly about turbines developed by Naval Energies' OpenHydro subsidiary

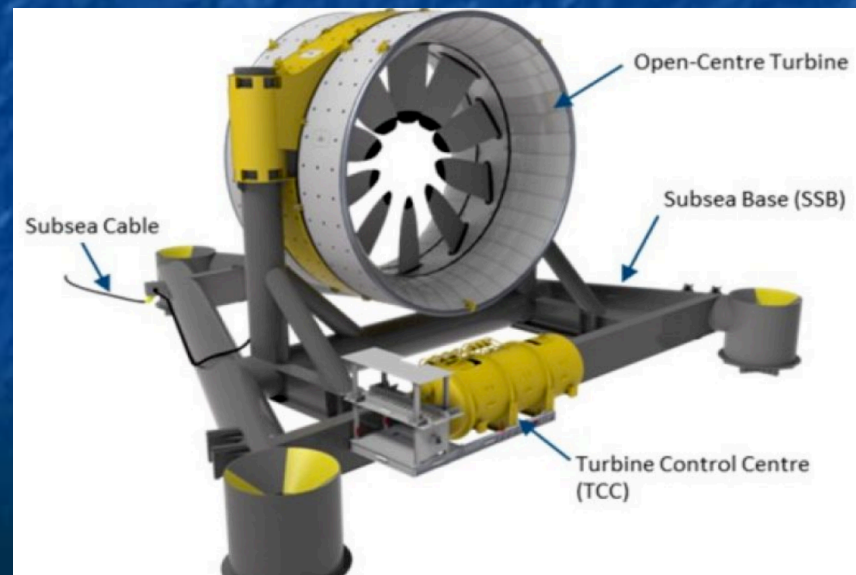
OpenHydro's turbines **were** compact, foundation-less, and bottom-clinging

But rather than connecting turbine blades inward to a central shaft

OpenHydro's turbine blades connected outward to a spinning ring

This distributed the tidal flow's force across different parts of the turbine

But it required more / larger moving parts, all in contact with corrosive seawater



The first OpenHydro prototype was lowered into the Bay of Fundy in 2009:



Figure:
<https://www.greentechmedia.com/articles/read/a-big-setback-for-tidal-power>

The scale of this **1 MW prototype** is seen by comparing its lifting rig's guard rails

The Bay of Fundy destroyed this turbine's blades within three weeks ¹⁻⁶

This sent OpenHydro "back to the drawing board,"

resulting in a 2 MW second generation turbine design,

of which two were scheduled for Bay of Fundy deployment in 2016

1) <https://fundyforce.ca/resources/f1c21770b59114114866df4d491b4dc2/Appendix1-2019-Cape-Sharp-Tidal-Venture-Monitoring-Report-FORCE.pdf>

2) <https://fundyforce.ca/about-us>

3) <https://www.cbc.ca/news/canada/nova-scotia/failed-tidal-turbine-explained-at-symposium-1.1075510>

4) <https://www.nationalobserver.com/2017/06/06/analysis/tidal-power-bay-fundy-dream-without-danger>

5) <https://www.greentechmedia.com/articles/read/a-big-setback-for-tidal-power>

6) <https://warktimes.com/tidal-timeline/events-of-2010/>

The first of these 2MW turbines was deployed on 7 November 2016



Figure:
<https://www.halifaxexaminer.ca/featured/losses-from-the-failed-tidal-generation-project-continue-to-mount/>

This turbine's increased size is again judged by comparing its barges' guard rails

Detailed information is very hard to find, but combined news reports¹⁻⁴ indicate that

not long after deployment, the force of the tides was so strong

that the turbine, on its triangular platform, was dragged along the seabed

This apparently led to the chain of a platform anchor becoming entangled

with the turbine's power output and sensor/control cables, causing it to fail

1) <https://fundyforce.ca/resources/f1c21770b59114114866df4d491b4dc2/Appendix1-2019-Cape-Sharp-Tidal-Venture-Monitoring-Report-FORCE.pdf>

2) <https://fundyforce.ca/about-us>

3) <https://www.saltwire.com/news/local/timeline-tidal-power-in-the-minas-passage-224062/?location=annapolis-valley>

4) <https://www.halifaxexaminer.ca/featured/losses-from-the-failed-tidal-generation-project-continue-to-mount/>

While that turbine lay abandoned, OpenHydro modified its sister 2 MW turbine

Those modifications continued for two years

During which **legal battles raged over recovery of the abandoned turbine**

The sister 2 MW turbine was finally lowered to the Fundy seabed on 22 July 2018

And its power output and sensor/control cables connected two days later ^{1, 2}

But only two days after that, OpenHydro's mother company Naval Energies

announced it was **abandoning tidal power and shutting down OpenHydro** ¹

When, in early September, the new turbine was briefly reconnected,

it was found that its **turbine had for some reason ceased spinning** ^{1, 3}

Eventually, as a part of lawsuit investigations, the Supreme Court of Nova Scotia

determined that in September the turbine had been **"damaged beyond repair"** ⁴

1) <https://fundyforce.ca/resources/f1c21770b59114114866df4d491b4dc2/Appendix1-2019-Cape-Sharp-Tidal-Venture-Monitoring-Report-FORCE.pdf>

2) <https://fundyforce.ca/about-us>

3) <https://globalnews.ca/news/4461039/team-investigating-why-rotor-not-turning-on-cape-sharp-tidal-turbine/>

4) <https://globalnews.ca/news/4644316/cape-sharp-tidal-turbine-damaged/>

As of 2019, there is a new Tidal Stream player in the Bay of Fundy:

Sustainable Marine Energy (SME), which deployed this turbine platform: ^{1, 2}



Figure:
<https://renews.biz/54496/sme-gets-nova-scotia-tidal-green-light/>

It's the first of three planned platforms - which seem to represent a step backwards:

The platform's four rear turbines appear to have two very conventional blades,

And they each look to be no more than three meters in diameter

Indeed, with the whole platform rated at only 0.420 MW, ² individual turbine capacity

must be only 0.105 MW = 1/15th that of MeyGen, 1/20th that of OpenHydro

1) <https://www.cbc.ca/news/canada/nova-scotia/sustainable-marine-energy-minas-tidal-lp-bay-of-fundy-tidal-power-1.5304276>

2) <https://sustainablemarine.com/news/pembaq-project>

Bottom Lines regarding Tidal Stream Power:

Necessarily fast tidal flows are much more common than at least I had expected:

This is due to funneling of tides within narrowing bays or straits (a.k.a. "firths")

and / or subsequent tides re-enforcing one another via Tidal Resonance

Based on such flows, studies now suggest that Tidal Stream

could provide a good fraction of at least a smallish nation's power ¹

Early vulnerable / shipping-interfering Tidal Stream towers have now been superceded

by far more plausible "compact, foundation-less, and bottom-clinging" designs

The largest of these new turbines have power production capacities not far below

that of today's very commercially successful wind turbines (i.e., ~1/8 to 1/4 as big)

However, turbine numbers are miniscule compared to what will be required for impact

Further, one of two large turbine designs failed almost immediately in real-world testing,

and the other has thus far survived barely 1/10th of the 20+ years likely

to be necessary for cost-effective large-scale power production

1) I hope my MacBane ancestors take no offense at my use of "smallish" in referring to their possibly-soon-to-be independent nation

Wave Power

Waves seem to be everywhere, and they cycle up & down so very often!

Suggesting that they're almost like a super Tidal Barrage:

One that cycles once every wave period - rather than once every ~12 hours

And one that spans vast areas - rather than just a paltry few dozens of km²

But that resemblance is deceptive:

Waves DO cycle specific volumes of water up & down,

but they do so by passing power **onward** with the moving wave

Thus, if an ideal converter captured ALL of a wave's energy as it crossed a line

there would be no energy left to drive waves onward beyond that line

(leaving the water surface **behind** that converter absolutely wave-free)

So wave energy will NOT be harvestable across vast areas

With realistic converters, it will instead be harvested when incoming waves

try to move through long obstructing **bands of those converters**

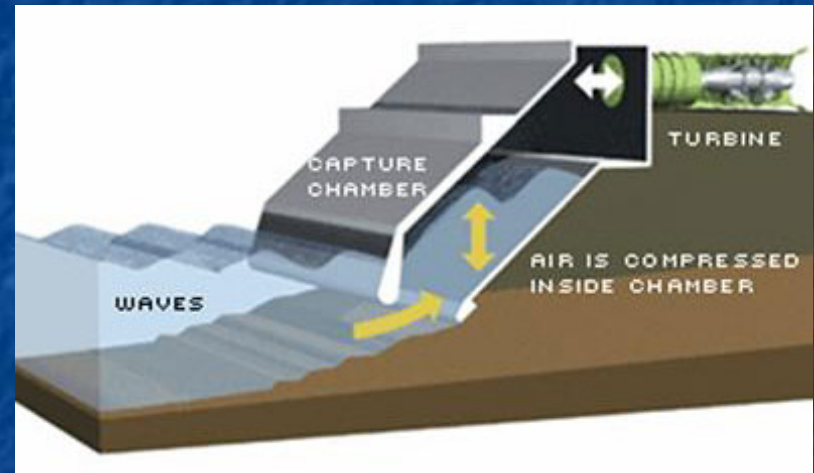
Wave Power converter prototypes & concepts:

Keeping moving parts & generators **OUT** of corrosive saltwater makes a lot of sense:

As done via these shoreline converters developed & tested at Brazil's **Coppe Subsea Technology Lab** ¹



Or as would occur with this shoreline conversion scheme proposed by the U.S. National Renewable Energy Lab (NREL) ²



But to produce a LOT of power, these would have to stretch along our coasts, completely occupying (and desecrating) the shorelines that so many of us cherish

1) http://www.bluebird-electric.net/wave_power_energy_generation.htm

2) https://openei.org/wiki/Wave_Energy

This scheme would instead position converters offshore:

Pelamis Wave used hydraulic pistons inside flexing joints to capture wave power

Which, unfortunately, DID submerge both pistons & joints in corrosive seawater

One prototype was built for the European Marine Energy Center, Orkney Scotland

A second prototype was built for Scottish Renewables

But after delivering only those two prototypes in 2010, the company folded ¹

And their vision of offshore bands of such serpents (right) was left unrealized:

!) https://en.wikipedia.org/wiki/Pelamis_Wave_Energy_Converter



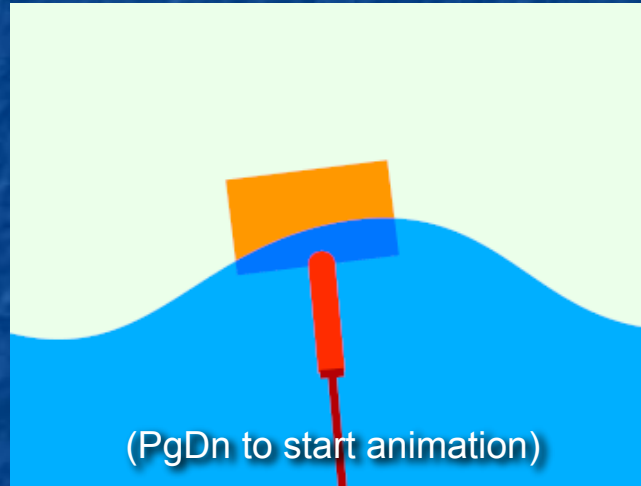
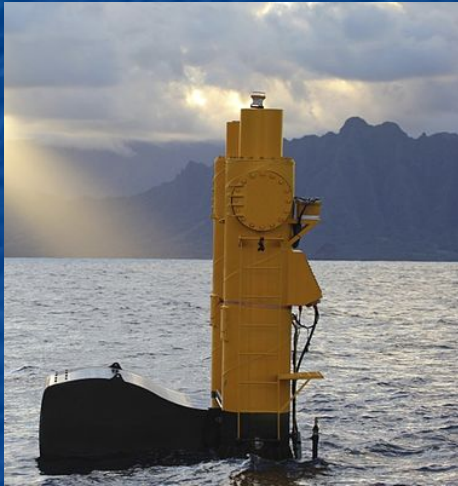
<http://www.biggreensmile.com/green-glossary/wave-power.aspx>



<http://www.biggreensmile.com/green-glossary/wave-power.aspx>

Other offshore schemes would use tethered buoys:

Hydraulic pistons, in cylinders above the buoy (and thus above the seawater), are pulled downward on each wave crest, producing electrical power (center) ¹



Northwest Energy Innovations produced the 20 KW "Azura" buoy (left) & promised a 0.5 MW successor by 2017 (which is no longer mentioned on their website) ²

National Geographic covered **Ocean Power Technologies'** 2014 buoy (right) ³
but wave power is now (also) barely discussed on that company's website ⁴

1) Center: [https://en.wikipedia.org/wiki/Azura_\(wave_power_device\)](https://en.wikipedia.org/wiki/Azura_(wave_power_device))

2) Left: <http://azurawave.com/projects/hawaii/>

3) Right: <https://www.nationalgeographic.com/news/energy/2014/02/140220-five-striking-wave-and-tidal-energy-concepts/>

4) <https://www.oceanpowertechnologies.com/about>

The **mWave** converter would instead reside on the sea bottom ¹

There (like newer Tidal Stream units) it would be less threatened by passing ships

But rather than exploiting the **water motion within** water waves,

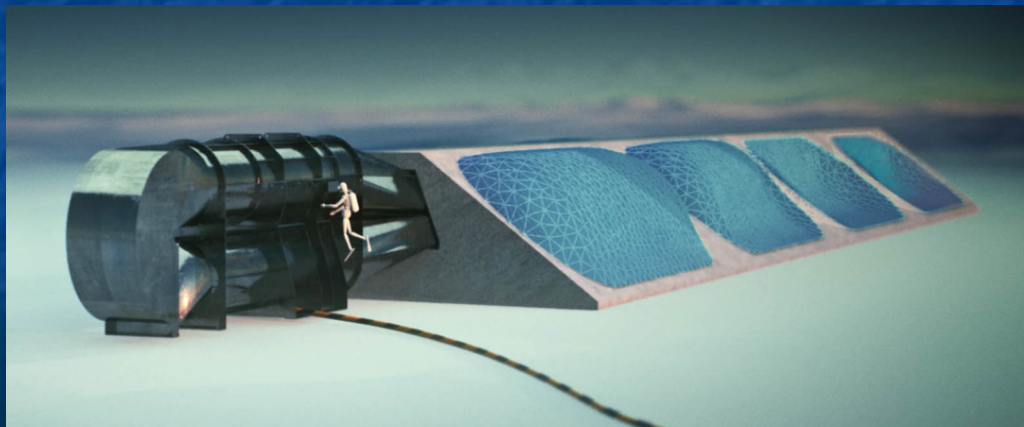
it would exploit the weight (and hence cycling pressure) of waves passing above

Within its reef-like structure, an air-filled **flexible** rubber bladder

is connected to an **inflexible** air tank

The increased water pressure below each passing wave then

squeezes air from the bladder, through a turbine generator, to the tank



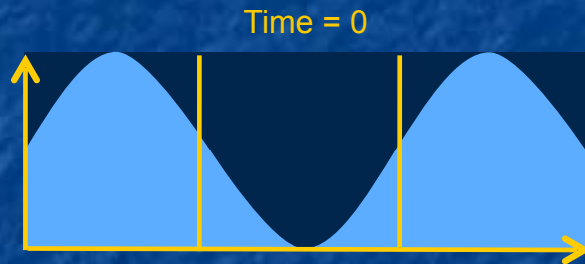
1) <https://www.bomborawave.com/mwave/>

Figure Wikipedia's Wave Power: https://en.wikipedia.org/wiki/Wave_power

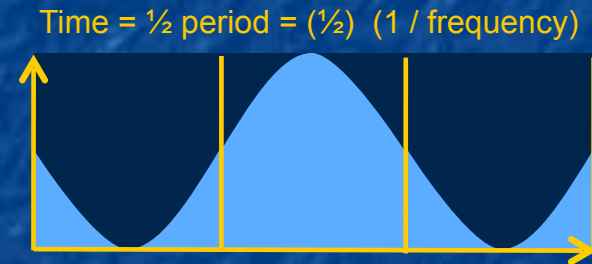
What is the MAXIMUM power that waves might provide?

We can estimate this based on the flow of gravitational potential energy:

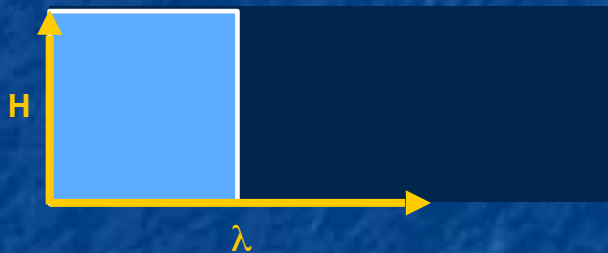
Wave at one point in time:



And half a wave period later:



To simplify this estimation, approximate the Time = 0 wave as being rectangular:



Squared off it has a **height of H**,

a **width of λ** (the common symbol for its **wavelength**)

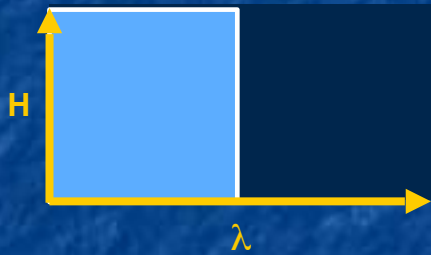
and a **length of L** (its perpendicular extension)

From earlier in this note set, each unit of water has gravitational potential energy:

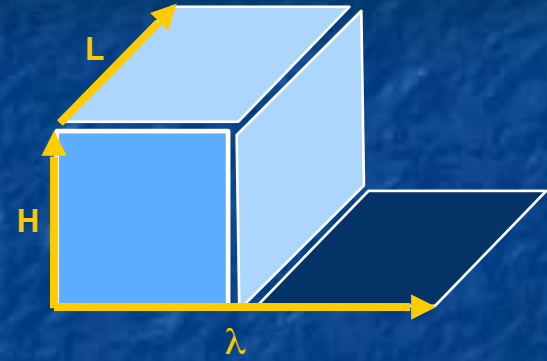
$$E_{\text{gravity}} = M g h = (\rho_{\text{water}} \text{ Volume}) g h = \rho_{\text{water}} \text{ Volume } h$$

Mapping the wave's movement in three dimensions:

T = 0 wave with this cross-section:



Becomes this 3D shape:



One wave period later, that wave is "gone" (shifted by λ out of frame to the right)

But its **water molecules have NOT in fact moved λ to the right**

they've instead fallen downward,

surrendering their gravitational energy,

which is then transferred forward out of frame to the right,

where it is reinvested into **new** water molecules,

which rise to form the wave as we see it one period later

How much gravitational potential energy moves forward from the $T = 0$ wave?

The wave's **total volume of water** = $V_{\text{total}} = H (\lambda / 2) L$

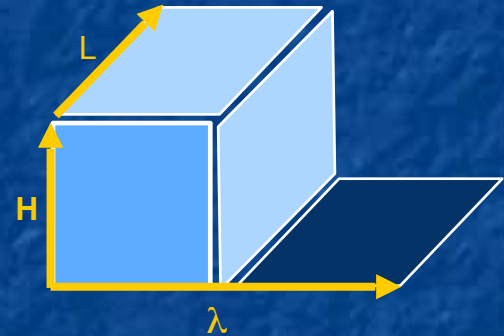
Gravitational potential energy of that volume of water = $\rho_{\text{water}} g \text{ Volume } \Delta h$

The fall distances (Δh)? As in our Tidal Barrage calculation:

Water at the top of the bar falls H

Water at the bottom of the bar falls 0

Giving an average fall of $H / 2$



Total gravitational potential energy lost (and transferred forward):

$$\Delta E_{\text{gravity}} = \rho_{\text{water}} g (\text{Total Water Volume}) (\text{Average fall}) = \rho_{\text{water}} g (H \lambda L / 2) (H / 2)$$

$$\Delta E_{\text{gravity}} = (1/4) \rho_{\text{water}} g H^2 \lambda L$$

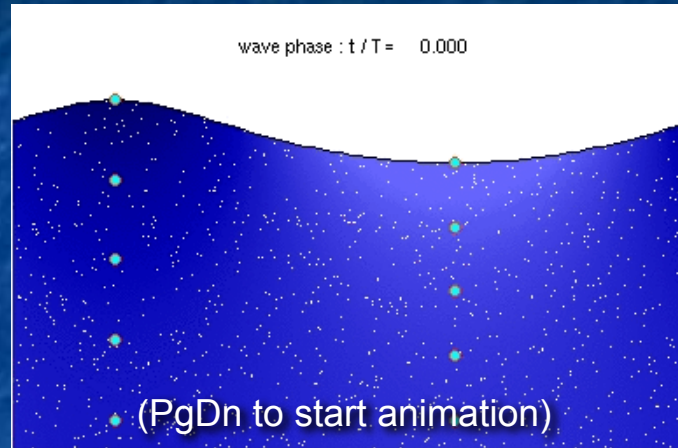
Wave power = That energy / time to move one wavelength = $\Delta E_{\text{gravity}} \times (v_{\text{wave}} / \lambda)$:

$$\text{Power of Wave} = (1/4) \rho_{\text{water}} g H^2 v_{\text{wave}} L$$

= Basis for estimating total wave power striking a coast

More complex models refine this equation (but the end result is unchanged)

One refinement acknowledges that the water in waves actually swirls in circles:



From Wikipedia on Wind Waves:

https://en.wikipedia.org/wiki/Wind_wave

Our formula already accounts for the energy of vertical water movement

But it should also account for the energy of horizontal water movement

An obscure bit of Physics, the **Equipartition Theorem**,¹ says that

Nature puts equal energy into both movements

That **DOUBLES** the **energy** within a volume of wave water

Which then **DOUBLES** the **power** transferred forward by the wave

1) See, for instance Wikipedia's write up: https://en.wikipedia.org/wiki/Equipartition_theorem

But another obscure bit of physics then halves that energy:

It says that wave energy does not move forward with the wave crests

It instead moves forward at **HALF** of the velocity of the wave crests

That's illustrated in this mathematical simulation of moving groups of waves:



(PgDn to start animation)

From Wikipedia on Group Velocity: https://en.wikipedia.org/wiki/Group_velocity

The wave groups (and their energy) move forward as the green dots

But wave crests move forward through those groups (as the speedier red dots)

This slower rate of energy movement **HALVES** the power delivered by waves

EXACTLY COUNTERING the previous slide's **DOUBLING**, taking us back to:

$$\text{Power of Wave} = (1/4) \rho_{\text{water}} g H^2 v_{\text{wave}} L$$

Some other bits of obscure physics are more useful:

Water waves slow as they enter shallower water. Why?

Because ocean bottom increasingly impedes the wave's swirling water motion

This increasingly bunches up the waves (i.e., shortens their wavelength)

Which, to conserve energy, demands that those waves become higher

NOTE: This is why low but extremely broad deepwater waves become

extremely high (but narrower) Tsunami waves as they move onshore

This would greatly complicate our calculation of wave power if not for the fact that,

while waves change shape moving TOWARD a shore,

they continue carrying the same power

(at least until they begin to break up and/or reflect AT that shore)

So all we really need is information about the waves as they **start out in deepwater**

Physics provides one such bit about deepwater waves:

In deepwater, waves move at a velocity of: $v_{\text{wave}} = g (\text{Wave Period}) / 2 \pi$ ¹

But Wave Period = The time interval between passing wave crests, thus:

(Wave Period) = (Wavelength) / (Wave Velocity) Or, in the common symbols:

$$T = \lambda / v_{\text{wave}}$$

Substituting this into the deepwater equation at the top of this page:

$v_{\text{wave}} = g (\lambda / v_{\text{wave}}) / 2 \pi$ Solving this for v_{wave} :

$$v_{\text{wave}} = (g \lambda / 2 \pi)^{1/2}$$

Substituting that into our earlier Wave Power equation:

Power of Wave = $(1/4) \rho_{\text{water}} g H^2 [g^{1/2} \lambda^{1/2} / (2 \pi)^{1/2}] L$

$$\text{Power of Wave} = (1/ 32\pi)^{1/2} \rho_{\text{water}} g^{3/2} \lambda^{1/2} H^2 L$$

All we now need is "typical" wavelengths (λ 's) and wave heights (H 's)

You can easily find such wave information online:

From Wikipedia's **Sea State** webpage: ¹

Waves 1.25 to 2.5 meters high are considered "moderate" (Sea State 4)

From Wikipedia's **Wind Wave** webpage: ²

1.5 meter high waves typically have a wavelength of ~ 33.8 meters

Allowing me to calculate the power of "moderate" waves with: $H = 1.5\text{m}$ & $\lambda = 33.8\text{m}$

$$\text{Power of "moderate" Wave} = (1/32\pi)^{1/2} \rho_{\text{water}} g^{3/2} \lambda^{1/2} H^2 L$$

$$= (1/32\pi)^{1/2} (1000 \text{ g / m}^3) (9.8 \text{ m / s}^2)^{3/2} (33.8 \text{ m})^{1/2} (1.5 \text{ m})^2 L$$

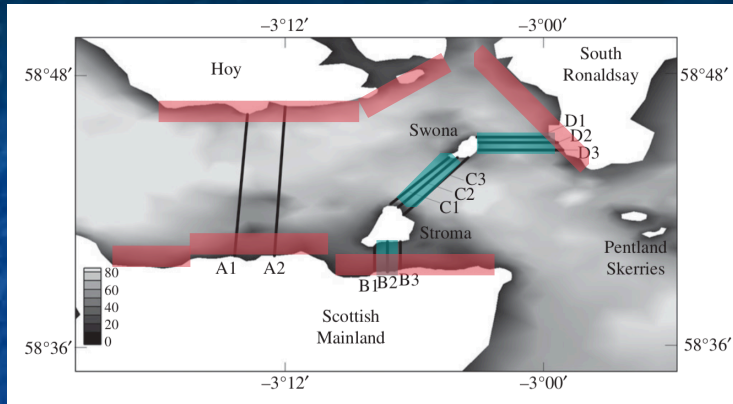
Power of "moderate" Wave: ~ 110 (kW / meter) L

where L is the length of coastline struck by "moderate" (1.5m x 33.8m) wave

1) Wikipedia on Sea State: https://en.wikipedia.org/wiki/Sea_state

2) Wikipedia on Wind Waves: https://en.wikipedia.org/wiki/Wind_wave

So COULD Wave Power compete in Pentland Firth or Hawaii?



Pentland Firth:

Green = Tidal Stream generators producing earlier study's "**up to 1.9 GW**"

Red = ~ 50 km long shoreline of the Firth

← ~ 20 km →

Using equation from the previous page, wave power incident on that shoreline is:

Maximum Pentland Firth wave power = (110 kW / m) (50,000 m) = **5.5 GW**

First answer: **YES**, Firth's wave power exceeds its tidal stream power

Might waves on Oahu's 365 km coastline ¹ satisfy its ~ 2 GW consumption? ²

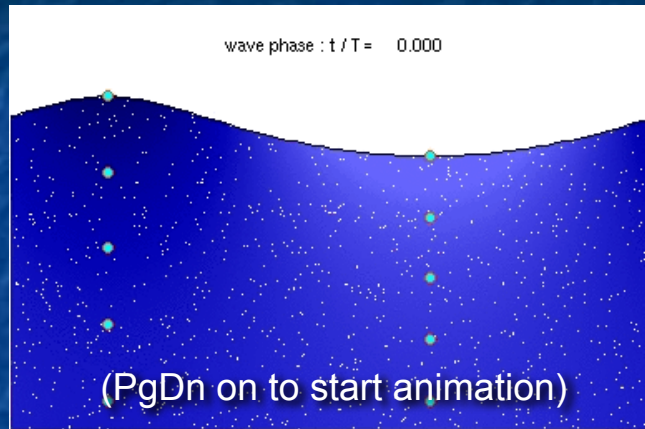
Maximum Oahu wave power = (110 kW / m) (365,000 m) = **40.1 GW**

Second answer: **EASILY**, at least with the RIGHT wave power technology

1) <https://www.hawaiiandreamvacations.com/about-oahu/about-oahu/>
2) Estimated from non-Maui data on: <https://www.hawaiianelectric.com/about-us/power-facts>

But what about pressure-driven schemes like mWave?

The above wave power equation offers little insight - but diagrams like this still do:



From Wikipedia on Wind Waves:

https://en.wikipedia.org/wiki/Wind_wave

They correctly indicate that water motion diminishes rapidly beneath the waves

But that implies lateral pressure differences also diminish rapidly beneath the waves

If not, their persistence would continue to drive lateral water movement

Imagine you were below a wave as its crest passed **directly** overhead,

its adjacent wave troughs would then be **almost** overhead

The deeper you were, the smaller the difference between **directly** and **almost**

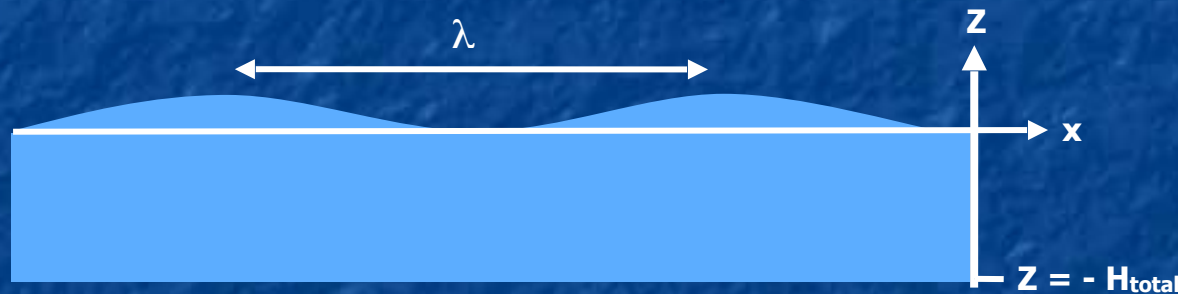
with pressure more and more reflecting **time-averaged** water height above you

My intuition thus suggested pressure waves would damp out quickly

With amplitudes likely plummeting at depths greater than one water wave wavelength

To check on my intuition, I dug up class notes from an MIT **Hydrodynamics** class ¹

The notes analyzed this situation of a wave moving atop a sea of depth H_{total} :



It took seven pages of physics + mathematics to finally come up with an equation describing the pressure wave at a depth z below such a water wave

That equation was the product of two terms:

The first term gave the pressure wave at the base of the wave (i.e., at $z = 0$)

The second term scaled that pressure wave downward at depths below $z = 0$

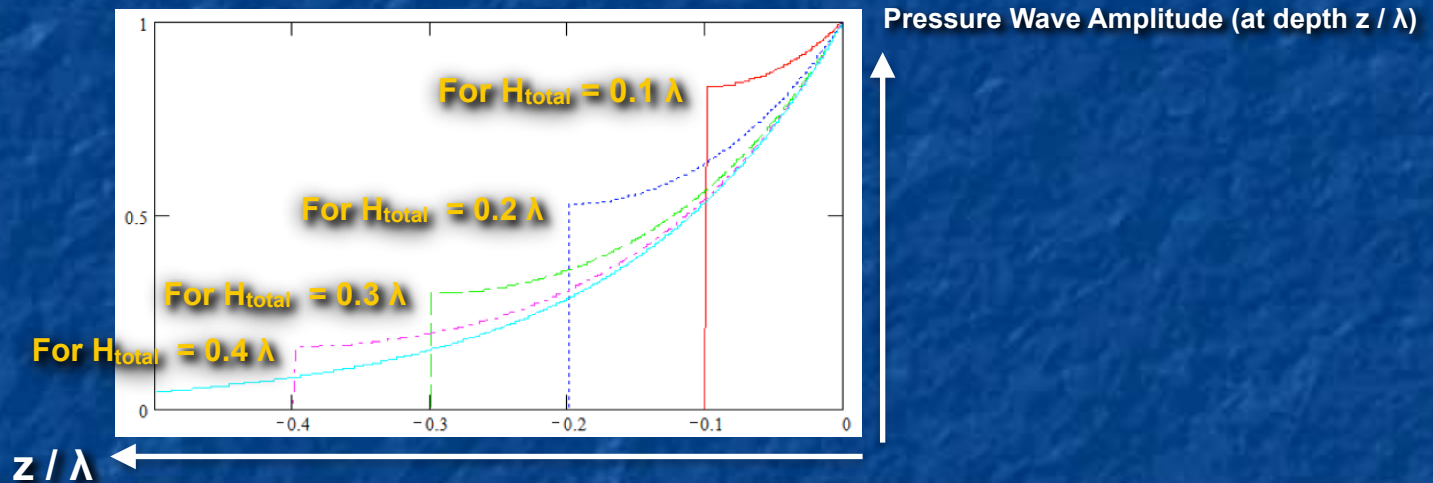
Using "Mathcad" software, I plotted out the scaling term: 1, 2

For that plot, I expressed both z (the observer's depth) & H_{total} (the total ocean depth) in units of λ (i.e., in multiples of the water wave's wavelength)

Here are my plots for seas with total depths of 0.1λ , 0.2λ , 0.3λ , 0.4λ and 0.5λ

Per my intuition, pressure wave amplitudes DO fall rapidly with depth (to the left)

Indeed, they damp out at even depths \ll one water wave wavelength



But the plots also showed something that I did NOT expect:

Damping is much more gradual in shallower seas (e.g., the sea 0.1λ deep)

*Consequences for mWave-like **submerged pressure wave** converters?*

It's not only desirable but **extremely desirable** that they be placed close to shore,
where water depth is only a small fraction of the local water waves' wavelength

But from above, typical wavelength for a "moderate" / "Sea State 4" wave is ~ 34m

A small fraction of that (e.g., less than 1/4) would be only < 8m (< 24 feet)

At that depth, normal water movement (and thus force) might be withstood

But **storm wave** movement / force might easily demolish submerged converters

While at the surface, storm water movement / force would be even **more** severe
making floating wave power converters even **more** susceptible to damage

Wave Power (of all kinds) thus confronts at least two severe challenges:

The protection of **any submerged moving parts** from seawater corrosion

The protection of **all parts** from extreme weather conditions

Bottom lines regarding Wave Power:

Contrary to at least **my** expectations:

Calculations show that wave energy incident upon our coasts is

large enough that it **could** someday provide much of our coastal power

But conversion schemes explored to this point seem to be:

Impractical

Prone to unacceptably early failure

Easily damaged by or damaging of the Environment

Which brings to mind a statement made by multiple sources:

"Wave Power seems to be where Wind Power was 30 years ago"

Which is to say:

Wave Power's huge potential is finally becoming appreciated

All we've got to do now is figure out **HOW** to do it!

Floating Photovoltaic Farms

One final form of water-based power:

Which uses no-longer-exotic solar power, but in an **EXOTIC LOCATION**:

On platforms floating out in the middle of smallish Lakes



<https://www.scientificamerican.com/article/putting-solar-panels-on-water-is-a-great-idea-mdash-but-will-it-float/>



<https://news.energysage.com/floating-solar-what-you-need-to-know/>

Why go to such trouble? Because, as noted in my evaluation of Tidal Stream power:

Solar farms produce low power per land area: $12\text{-}50 \text{ W} / \text{m}^2 = 12\text{-}50 \text{ MW} / \text{km}^2$

Further, unlike wind farms, they require **EXCLUSIVE USE of the land they occupy ¹**

1) See my note set: *Today's Photovoltaic Solar Cells* ([pptx](#) / [pdf](#) / [key](#))

But open land is often not available near our cities & factories

Or, if available, those cities & factories have probably driven up its expense

So the idea is to instead put small arrays of solar cells on floating platforms

Which might be no more than slightly-reinforced slabs of styrofoam

And to then tie those individual platforms into flexible rafts

With ties possibly provided by the solar cell output cables themselves

But why limit such floating solar farms to only "smallish" lakes?

Wouldn't LARGE LAKES offer the opportunity to build LARGE floating farms?

For instance, around the edges our U.S. Great Lakes

Ecological impact on such lakes is of course a concern

But there's also the issue of Water Wave power (as discussed above):

On any large body of water, winds can produce large waves

which might EASILY devastate fragile solar cells & floating platforms

The potential of FPV (Floating Photovoltaic) power?

Just a few years ago it seemed (at best) not much more than a curiosity

If so, it's a curiosity that has now found its niche:

Its claimed that in 2016-17, 200 MW of FPV power was installed

And that by the end of 2018 that it had reached 1.1 GW ¹

That remarkable growth is attributed to factors including: ¹⁻⁴

- Costs only 25% higher than land-based solar due to:

Elimination of land costs

Elimination of foundations & supporting structures

Elimination of often otherwise required artificial PV cell cooling schemes

Simple / inexpensive construction & decommissioning (float out / float off)

- Massive (up to 80%) reduction in evaporation from water storage reservoirs

- Elimination of algae blooms (and resultant fouling) in industrial cooling ponds

1) https://en.wikipedia.org/wiki/Floating_solar

2) <https://www.solarpowerworldonline.com/2019/07/will-floating-solar-arrays-float-or-sink/>

3) <https://www.scientificamerican.com/article/putting-solar-panels-on-water-is-a-great-idea-mdash-but-will-it-float/>

4) <https://news.energysage.com/floating-solar-what-you-need-to-know/>

*Global warming makes FPV's suppression of reservoir evaporation **very** attractive*

But FPV's use in cooling ponds also has very substantial potential,

especially in the cooling ponds already frequently associated with power plants

A particularly large example: Virginia's largest lake, 53 km² Lake Anna, ¹

which is actually the manmade "cooling pond" for a dual nuclear power plant:



The possibility:

From a few slides ago:

Solar PV farms = 12-50 MW / km²

Multiplying that by Lake Anna's 53 km²:

=> Possible PV plant of 636 - 2650 MW

The upper (likely optimistic) number actually

exceeds dual nuclear plants' 1790 MW ²

1) https://en.wikipedia.org/wiki/Lake_Anna

2) https://en.wikipedia.org/wiki/North_Anna_Nuclear_Generating_Station

Wind Power Balloons & Kites

Is there an analogous "EXOTIC LOCATION" awaiting wind power?

It wouldn't be out on the water because, for wind power, that is no longer exotic:

Near shore & floating-wind turbines are already mainstream technologies ¹

Flying Wind Turbines would still qualify as exotic - But why go to the trouble?

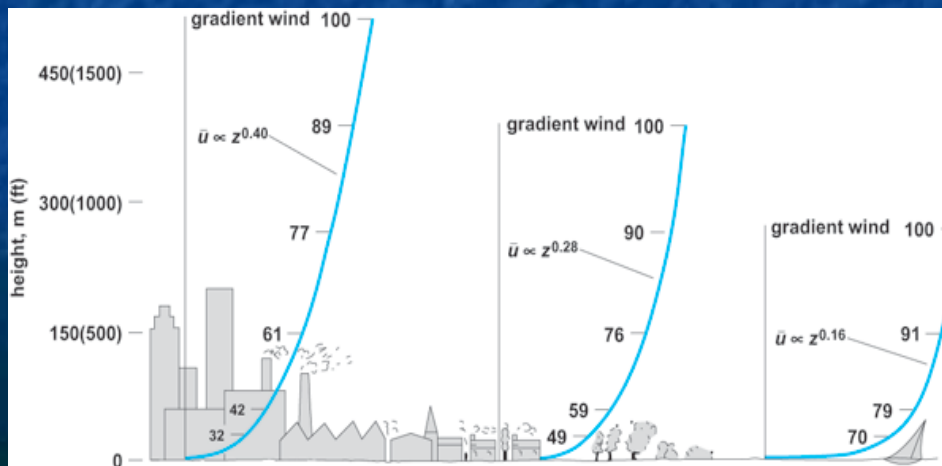
One reason: As with water flows, wind flow power increases as **velocity cubed**

Which makes reaching even modestly faster winds very desirable

Winds are ultimately driven by thermal gradients high in the atmosphere

And they are ultimately slowed by interaction with the ground (& things on it)

The net result: Wind speed increases rapidly above the ground: ^{2, 3}



1) See my notes set:
Wind Power - Part II ([pptx](#) / [pdf](#) / [key](#))

2) See my note set:
Wind Power I ([pptx](#) / [pdf](#) / [key](#))

Figure & 3) <http://rockets2sprockets.com/issue-cross-winds-wind-tunnels/>

The reach higher / faster winds, turbine towers have grown taller:

The logical progression (according to at least some entrepreneurs)?

Airborne wind turbines

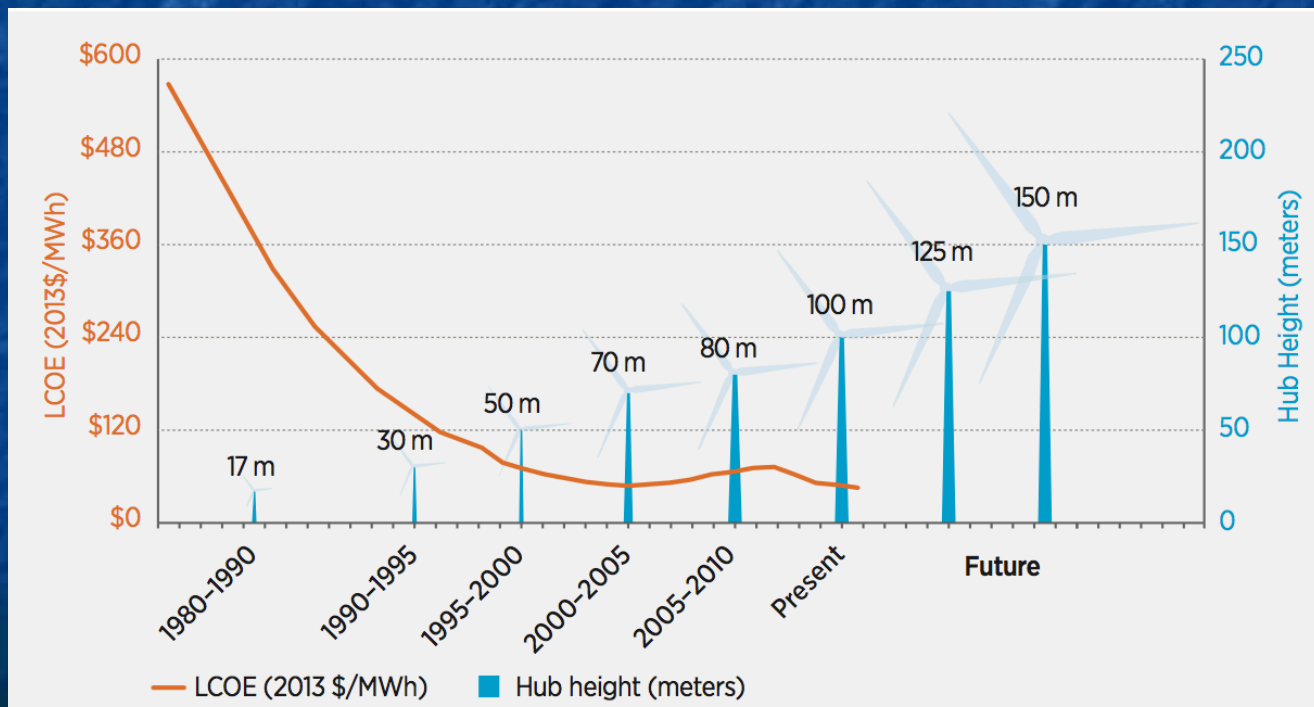


Figure: p 63. Wind Vision Report – US DOE: <https://www.energy.gov/eere/wind/wind-vision>

Which might actually look more like this:

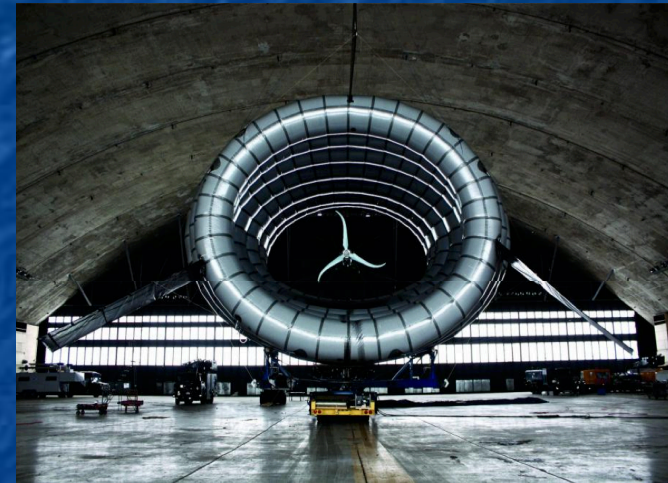
As featured in a 2014 Popular Science Magazine article: ¹

The Massachusetts startup company **Altaeros** was assembling a "Buoyant Air Turbine"

- A helium-filled cylindrical wing held aloft by the combination of helium's buoyancy + wing's lift
- To fly at up to 2000 feet, in winds up to 75 MPH

Prototype's Specifications (picture at right):

- Fourteen feet long
- Designed for **30 kW** power out
- Larger model to produce **200 kW** (with megawatt unit envisioned)



Target Markets?

- Remote sites with weak sunlight (=> grant from Alaska Energy Authority)
- Temporary industrial sites (e.g. construction or well drilling)
- Sites with low ground wind speeds

1) <https://www.popsci.com/article/science/quest-harness-wind-energy-2000-feet/>

Or it might be simpler (and safer) to just: **Go Fly a Kite**

As featured in a 2012 Spectrum Magazine article: ¹

The North Carolina company **WindLift** proposed getting rid of balloons:

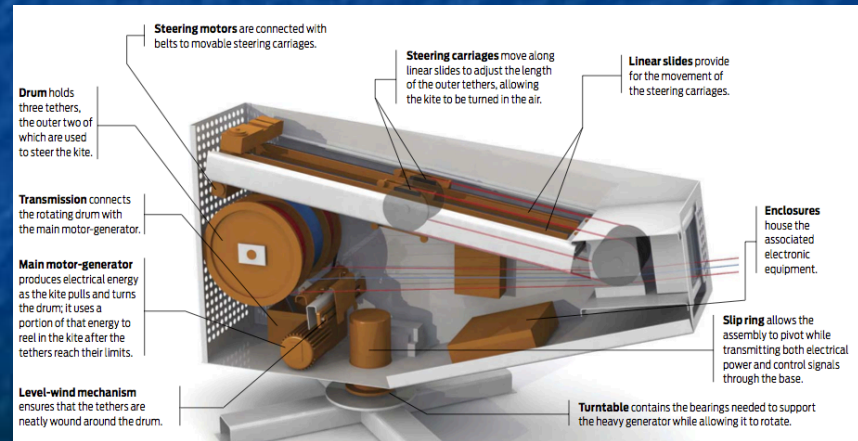
- Eliminating their very expensive helium
- Keeping the heavy electrical generator on the ground, thus decreasing required lift AND the mass available to fall on someone / something

Kite tugs out rope turning ground based generator (motor)

Motor (generator) then reels back in partially collapsed kite - then repeat cycle:



Prototype kite



Ground generator unit

1) <http://spectrum.ieee.org/energy/renewables/the-benefits-of-airborne-wind-energy>

*These designs **would** exploit faster high altitude winds*

But being so much smaller than present day ground-based wind turbines,

they would intercept vastly narrower wind area,

with probable result of **producing far less wind power per turbine**

Further, while Floating Photovoltaics operating in their "exotic location"

would produce power in the absence of human operators

Wind turbine **balloons & kites would require human monitoring during operation,**

plus likely intervention morning & evening to raise & lower them

It's thus unsurprising that **Altaeros's website no longer discusses wind power** ¹

Or that, while **WindLift continues to pursue that goal,** ²

its website now describes an **entirely different non-kite design,**

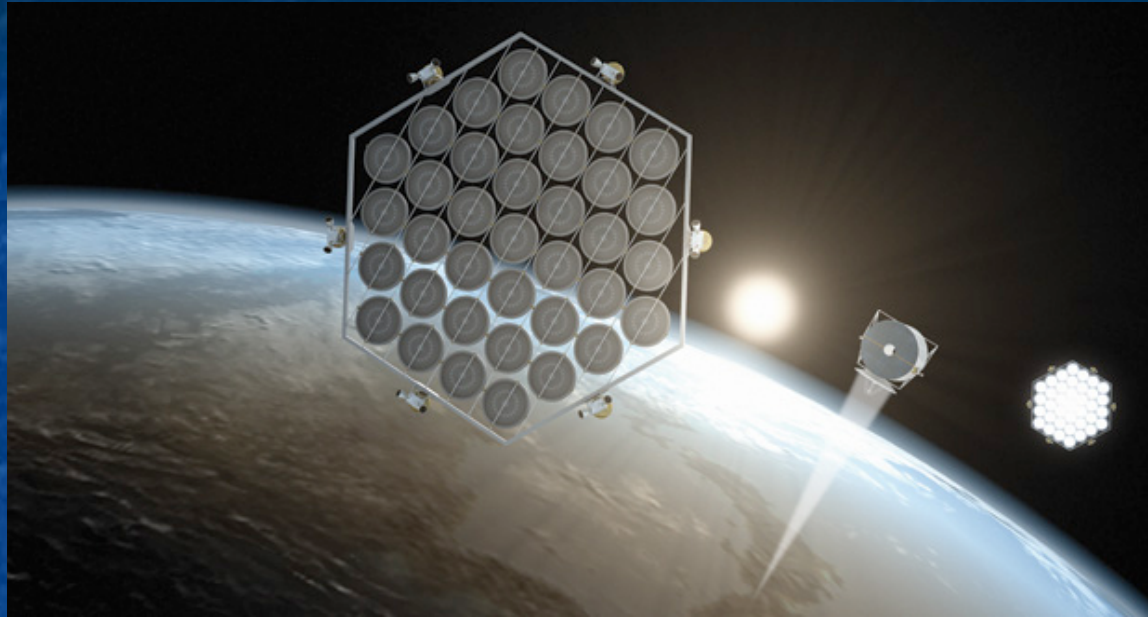
which appears to have now **"flown" in only computer simulations**

1) <http://www.alt aeros.com/>

2) <https://windlift.com/>

Solar Power Satellites

To have a much larger impact, what about: **Orbiting Solar Arrays?**



Which have been proposed by the Japanese Aerospace Exploration Agency (JAXA): ¹

Said to be possible **within twenty five years, with 1 GW power output**

Their power would be beamed down via microwave radio or laser beams

Array would weigh more than 10,000 tonnes and be several kilometers across

1) *How Japan Plans to Build an Orbital Solar Farm*, IEEE Spectrum Magazine, April 2014
online at: <http://spectrum.ieee.org/green-tech/solar/how-japan-plans-to-build-an-orbital-solar-farm>

Motivation (at least) is crystal clear:

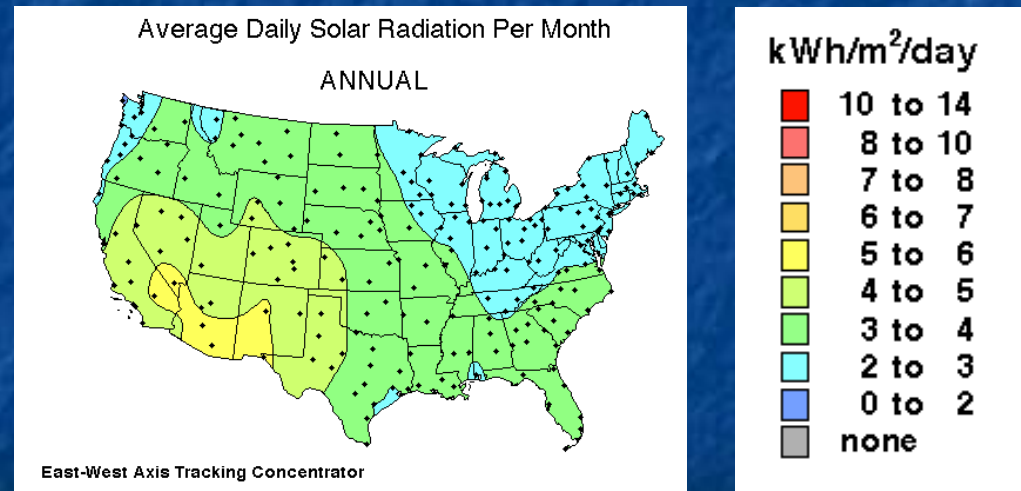
As described in my Solar Power note sets:

Our atmosphere absorbs $\sim 1/4$ of sun's power: $1.35 \text{ kW} / \text{m}^2 \Rightarrow 1 \text{ kW} / \text{m}^2$

Remainder is diluted when incident at shallow angles (i.e., not at noon)

And totally blocked by the earth itself (for a particular location) half the time

Net result (from U.S. National Renewable Energy Lab calculator website):



But $1 \text{ kW-h/m}^2/\text{day} = 41.6 \text{ W} / \text{m}^2$

So BEST U.S. sites have annual average incident solar power of $\sim 200 \text{ W} / \text{m}^2$

Versus an orbital solar array:

Once aimed at the sun, it should **stay** aimed at the sun (except for tidal effects)

And, when not blocked by the earth, a satellite receives that constant 1350 W / m^2

Almost 7X better than our BEST U.S. sites

And ~ 15X better than at our poorer (non-desert contiguous 48 state) sites

But the (first) big caveat is "when not blocked by the earth"

Time for a little orbital mechanics:

We want an object (the solar array) to orbit at a distance r above earth's center

Acceleration of object due to earth's gravity = $G M / r^2$

Inducing a centripetal acceleration on object = v^2 / r

Where $v = \text{orbital circumference} / \text{orbital period} = 2 \pi r / T$

Equating gravitational and centripetal forces to get orbits:

$$G M / r^2 = (4 \pi^2 r^2 / T^2) / r \quad \text{which yields} \quad \mathbf{(4\pi^2 / GM) r^3 = T^2}$$

$$G \text{ (universal gravitational constant)} = 6.67 \times 10^{-11} \text{ m}^3 / (\text{kg} - \text{s}^2)$$

$$\text{Earth parameters:} \quad M = 5.97 \times 10^{24} \text{ kg} \quad \text{Radius} = 6371 \text{ km}$$

$$\text{So earth's circumference} = 40,029 \text{ km}$$

(Which I remember as 24,000 miles => Equatorial spin of 1000 MPH)

$$\text{Constant } (4\pi^2 / GM) \text{ in equation then becomes: } 9.913 \times 10^{-14} \text{ s}^2 / \text{m}^3$$

Some space agency is going to have to launch pieces of solar array into orbit

Most launches are into **LEO** (low earth orbit) 160-2000 km above surface

ISS orbits ~ 400 km above earth => orbital radius of 6800 km, and a period of:

$$\mathbf{T = \sqrt{[9.913 \times 10^{-14} \text{ s}^2 / \text{m}^3 \times (6.8 \times 10^6 \text{ m})^3]} = 5,583 \text{ sec} = \mathbf{93 \text{ minutes}}$$

Problems with low earth orbit (LEO):

Earth will still block the sun half the time

We just lost half of our potential power enhancement!

Satellite won't stay above our location

Assuming that world is not willing to share in the cost & benefit of the satellite,

How do WE (the builders / financiers of the array) get all of its power?

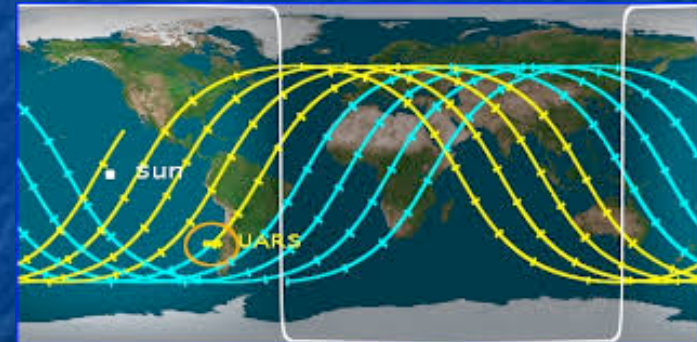
We'd have to store power until array passed overhead - which is NOT once per orbit!

It passes overhead far less frequently

Because the earth is rotating **under** its orbit:

Figure plots flyovers as ~ once in 8 orbits

=> twice a day (only!)



So we'd ALSO need **HUGE orbiting energy storage** (12 GW-hours of it!)

So instead use a geosynchronous orbit:

Meaning that we now want an orbital period of one day to match our rotation

Put $T = 24 \text{ hours} = 86,400 \text{ seconds}$ into $(4\pi^2 / GM) r^3 = T^2$ and solve for r :

$$r = [(8.64 \times 10^4 \text{ s})^2 / (9.913 \times 10^{-14} \text{ s}^2 / \text{m}^3)]^{1/3} = 42,227 \text{ km}$$

Subtracting out earth's radius => 35,856 km above earth surface

How much time will orbiting solar array then spend in earth's shadow?

Orbital circumference is now $2 \pi \times 42,227 \text{ km} \sim 265,000 \text{ km}$

Width of earth's shadow \sim earth diameter = $2 \times 6371 \text{ km} = 12,742 \text{ km}$

Fraction of time in shadow $\sim 12,742 / 265,000 \sim 4.8\%$

So we'd then get almost full 7X–15X enhancement of solar energy to the array!

But at least two very non-trivial issues remain:

First: The cost of launching array's parts into geosynchronous orbit:

NASA figure for cost to launch into (unspecified) orbit is \$10,000 / kg ¹

This, almost certainly, refers to low earth orbit only

Gravitational potential energy goes as $1/r$

r for high geosynchronous orbit is $\sim 6X$ r for low earth orbit

If cost scales as potential energy of orbit, geosynchronous cost $\Rightarrow \sim 60$ k\$ / kg

Planned Japanese station weighs 10,000,000 kg \Rightarrow **\$6 x 10¹¹ = 600 B\$** to launch

If array provided 1 GW (10⁶ kW) power for 20 years (limited by cell lifetimes):

Launch cost (alone) = \$6 x 10¹¹ / [(20 x 365 x 24 hours) x (10⁶ kW)]

To cover that cost (alone), charge for satellite's power would have to be:

3.42 \$ / kW-hour vs present day U.S. power costs of 10-20 cents /kW-h

(i.e., solar array's power would be 17 to 34 times more expensive)

1) http://www.nasa.gov/centers/marshall/news/background/facts/astp.html_prt.htm

The second issue: The array's 1 GW power beam:

Now only possible using microwaves (not lasers)

Which would be **aimed** at offshore receivers:

But microwave beams naturally spread out

And there is the nightmare scenario of a beam
being deliberately diverted as a weapon

Hard proof of RF radiation harm is as yet very slim

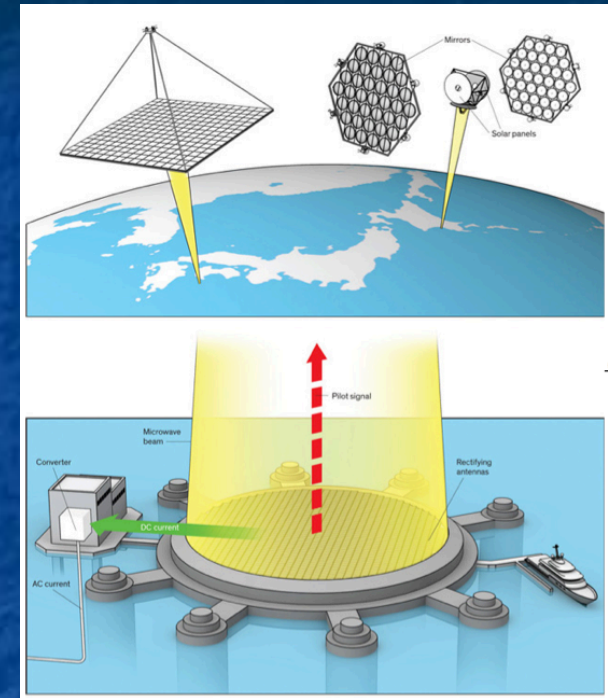
But many worry about cell phones & AC power lines

For which US / Euro power limits are currently:

1.6 - 2 W of RF radiation / kg of tissue ¹

A 1 GW beam could exceed that limit even spread over 500,000,000 kg of living tissue

I wouldn't want to go anywhere near the proposed power receiver (figure), and
I'd be uneasy with an unfriendly country's array orbiting anywhere above me



*How Japan Plans to Build an Orbital Solar Farm,
IEEE Spectrum Magazine, April 2014*

1) http://en.wikipedia.org/wiki/Mobile_phone_radiation_and_health

Fusion Power

What about the holy grail of clean power: **Nuclear Fusion?**

To start at the beginning, what exactly **is** nuclear fusion?

And how does it compare with conventional chemical fusion (i.e., chemistry)?

Chemical fusion combines atoms into molecules (or larger molecules)

Nuclear fusion combines atoms (or atom fragments) into larger atoms

Compare those alternatives for a pair of starting "Deuterium" atoms:

Deuterium = Hydrogen with a nucleus of 1 proton (●) **PLUS** 1 neutron (●)

(Starting with both protons + neutrons simplifies nuclear fusion)

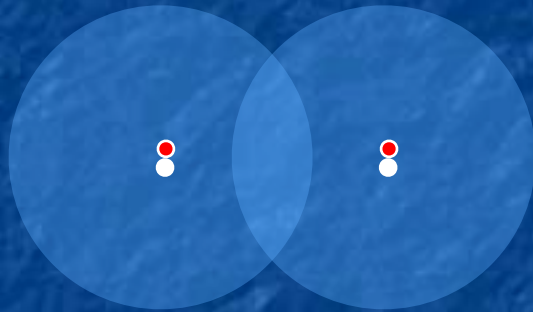
The Reactants:



The **products** of chemical fusion vs. nuclear fusion:

Chemically fused pair of Deuterium atoms:

(=> 1 Deuterium molecule)



$\sim 10^{-10}$ m

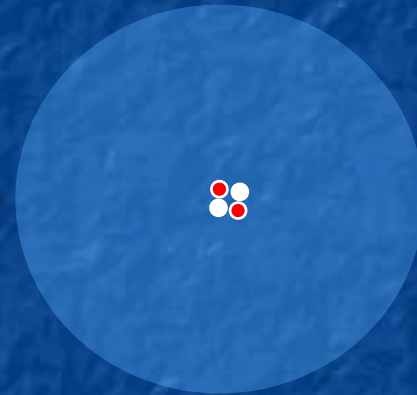
Energy liberated:

9.6 electron-Volts per molecule

=> **222** kcal per mole ¹

Nuclear fused pair of Deuterium atoms:

(=> 1 Helium atom)



$\sim 10^{-15}$ m

Energy liberated:

17.6 Million electron-Volts per atom ²

=> **409 million** kcal per mole

Nuclear Fusion's energy advantage: ~ 2 million times

PLUS the fact that its products are often benign simple atoms

1) https://atct.anl.gov/Thermochemical%20Data/version%201.118/species/?species_number=348

2) <http://hyperphysics.phy-astr.gsu.edu/hbase/NucEne/fusion.html>

The challenge is GETTING nuclear reactions to even occur

Chemical reactions require close approach of the reactant atom's electron clouds

A catalyst can facilitate this by getting those atoms to

settle down on its surface, immediately beside one another

Or in the absence of an effective catalyst, heat can be added to accelerate

reactants' motion to the point that collisions will drive them close together

Nuclear reactions require close approach of the reactant's nuclei

For intact atoms, negative electron cloud repulsion prevents such an approach

Instead, extreme heat must be added to the point that:

1) **Electron clouds are torn from nuclei** (creating what's called a **plasma**)

And that

2) Nuclei accelerate to speeds so fast that their positive nuclear charge

repulsion is overcome by the momentum / energy of their motion

How hot is hot enough?

That's determined by how **close** the speeding nuclei must come to one another

The answer (NOT taught in high school physics, or even most college physics classes):

Nuclei must come close enough for the **Strong Nuclear Force to kick in**

It's "strong" because it offsets the charge repulsion between the nuclei's protons

Fortunately, all we need to know about the **Strong Nuclear Force is that**

it only becomes significant at nuclear separations < 1 femtometer (10^{-15} m)

(i.e., as in my preceding figure, at dimensions comparable to the size of nuclei)

For Nuclear Fusion to begin, two speeding nuclei must have enough kinetic energy

that they can overcome the potential energy barrier created by

their charge repulsion at that critical separation of about 1 femtometer

Once / if they top that potential energy barrier, the **Strong Nuclear Force takes over,**

drawing those two nuclei together, fusing them into a new larger single nucleus

A "back of the envelope" calculation of that "ignition" condition:

From high school physics, for two atoms with n protons in each of their nuclei, the charge repulsion energy between those nuclei when separated by r is:

$$E_{\text{charge repulsion}} = (1/4 \pi \epsilon_0) (n q)^2 / r$$

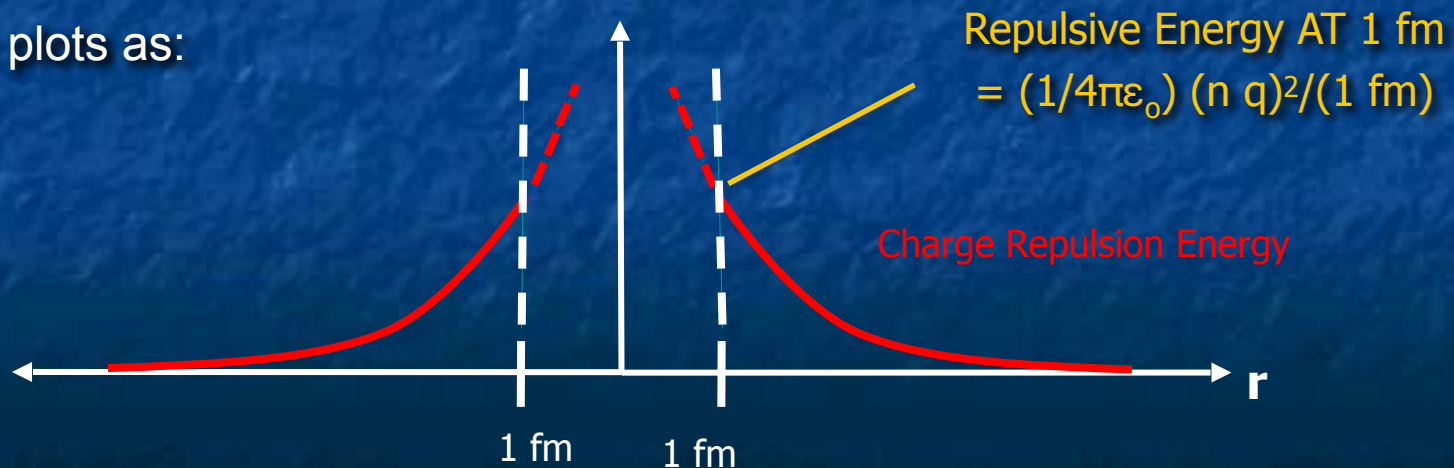
Where q = is magnitude of proton charge = 1.6×10^{-19} Coulombs

ϵ_0 = permittivity of free space = 8.85×10^{-12} Coulombs / Volt-meter

n = number of protons in each nucleus

r = separation of the nuclei

Which plots as:



*Nuclei must **start** toward one another with that much **kinetic energy***

A particle's kinetic energy is determined by its temperature

In fact they are virtually synonymous, with their relationship being

Kinetic energy of a particle (at temperature T) ~ k T

$$k = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ kg-m}^2 / \text{s}^2 \text{ } ^\circ\text{K}$$

To get the required temperature, equate that kinetic energy with the

repulsive barrier's height at 1 fm (where **Strong Nuclear Force** takes over):

$$k T = (1/4 \pi \epsilon_0) (n q)^2 / (1 \text{ fm}) \quad \text{Or:} \quad T = (1/4 \pi \epsilon_0 k) (n q)^2 / (1 \text{ fm})$$

Putting in the numbers for Deuterium nuclei (where number of protons $n = 1$):

$$T = (1.6 \times 10^{-19} \text{ C})^2 / (4\pi)(8.85 \times 10^{-12} \text{ C/V-m})(1.38 \times 10^{-23} \text{ kg-m}^2/\text{s}^2 \text{ } ^\circ\text{K})(10^{-15} \text{ m})$$

$$= 17 \text{ billion } ^\circ\text{K} \text{ (C-V / kg-(m/s)}^2\text{)} \quad \text{things in parenthesis} = \text{Joule/Joule} \Rightarrow 1$$

Temperature to initiate hydrogen fusion is ~ 17 BILLION degrees (K) ¹

1) Wikipedia's official / supposedly expert plots show onset of strong fusion at ~ 1-10 billion degrees Kelvin:
https://en.wikipedia.org/wiki/Nuclear_fusion

That is (literally) an otherworldly high temperature:

In fact, until 1 November 1952, it was only reached deep within **stars**

One barrier to Fusion is thus GETTING nuclei up to such stellar temperatures

Another barrier to Fusion is KEEPING nuclei at such temperatures

LONG ENOUGH enough for them to experience a chance collision

ANY CONTACT / INTERACTION WITH NORMAL MATTER (still a billion times cooler)

and they'll immediately lose energy to that matter, quenching their temperature

To accomplish this, in most experimental Fusion Reactors (the ONLY kind we have!):

- 1) Nuclei are suspended in ultrahigh vacuum chambers (which are ~ matter-free)
- 2) They're held away from chamber walls by specially-shaped magnetic fields
- 2) They're heated via pulsed electromagnetic fields

Confining those ultrahot nuclei via magnetic fields:

The necessary fields are created by strong electromagnets wrapped around the plasma/nuclei-containing ultrahigh vacuum chamber

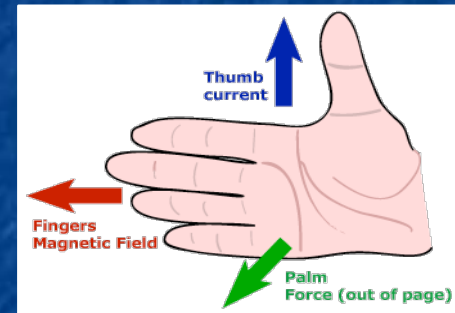
These create the magnetic field configuration known a "magnetic bottle"

As explained in my note set about **Electric and Magnetic Fields** ([pptx](#) / [pdf](#) / [key](#)):

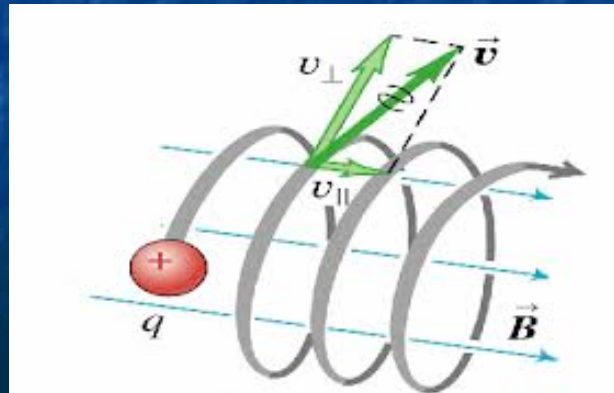
Magnetic fields push charges **sideways, without affecting their energy**

This strangeness is described by the "first right hand rule:"

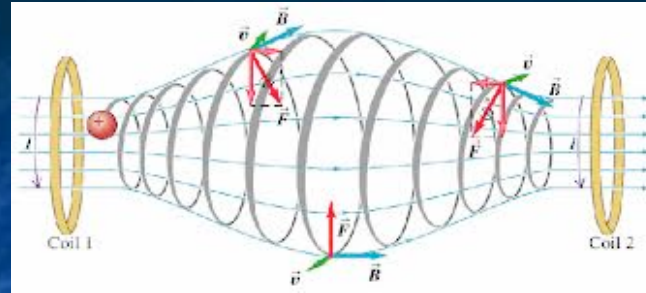
The result: Charge trying to **cross** Magnetic field lines instead ends up **looping around** or **spiraling along** those lines:



<http://www.swapyournotes.com/article/detail/article/detail.html/632/>



Early Magnetic Bottles were sealed by magnetic mirrors at both ends:

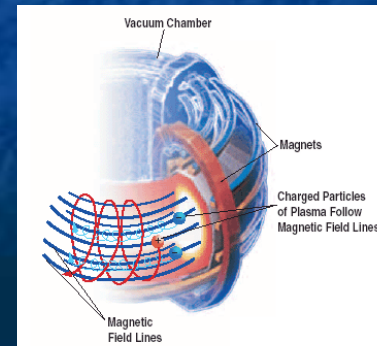
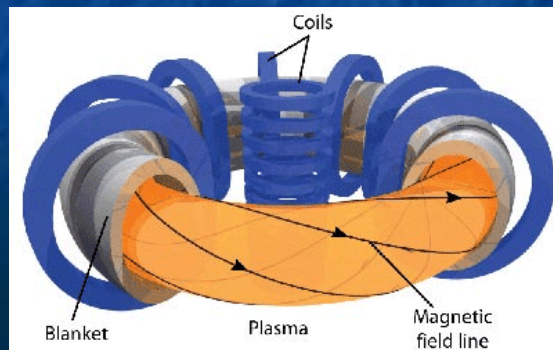


<http://astarmathsandphysics.com/a-level-physics-notes/electricity/a-level-physics-notes-the-magnetic-bottle.html>

The "mirrors" were just concentrated / intensified end magnetic fields which caused the nuclei to slow, and then reverse the direction of their spiral such that (most) went back and forth until (hopefully) a collision occurred

Work on such **Magnetic Confinement Fusion**¹ began in the **1940's**

The first major advance came in the 1960's with Russia's toroidal **Tokamak** in which nuclei could just **continue spiraling along loops** of magnetic field



<http://new.math.uiuc.edu/math198/MA198-2009/farrell1/>

http://ffden-2.phys.uaf.edu/211_fall2002.web.dir/paul_gradney/gradney_fusion/tokamak.htm

1) https://en.wikipedia.org/wiki/Magnetic_confinement_fusion

Over the last **75 years** non-magnetic schemes have **also** been tried:

Most notably **Internal Confinement Fusion** ¹ which **resembles a hydrogen bomb**

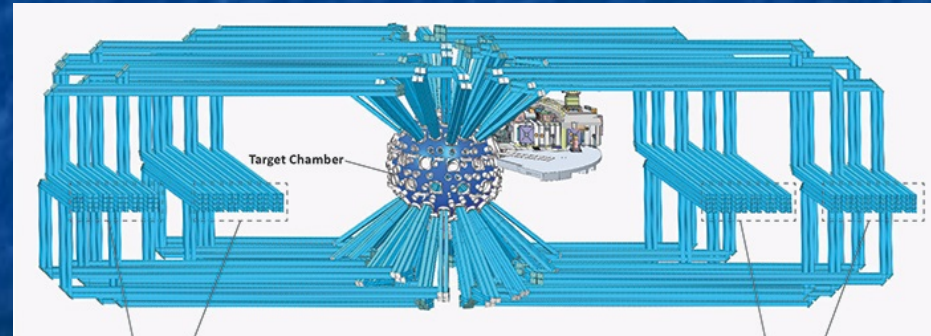
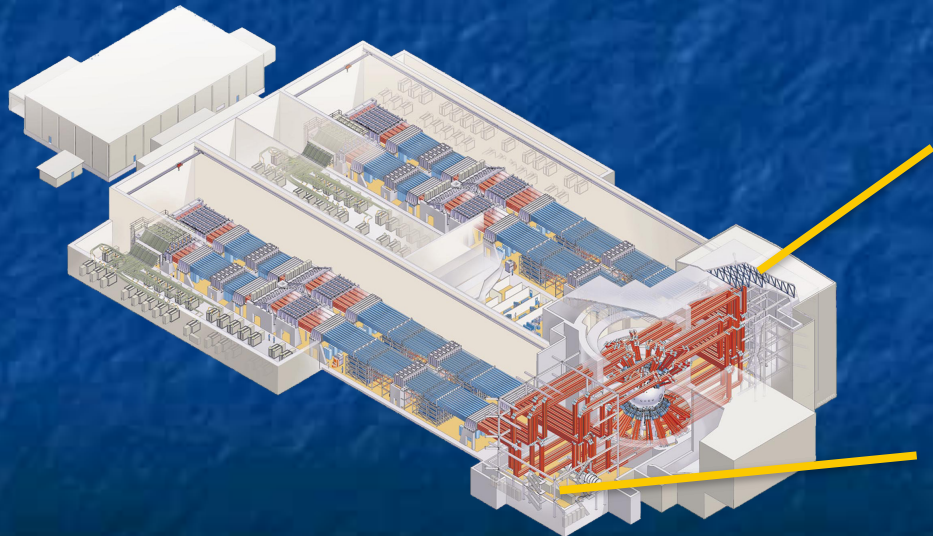
in its use of a surrounding explosive force to **implode** a nuclear charge

But in contrast, the quantity of nuclear fuel is extremely small (e.g., 10 mg) ¹

And implosions are extremely short (typically induced by ultrashort laser pulses)

Best known was U.S. Lawrence Livermore National Lab's **National Ignition Facility** ²

in which 192 ultrahigh power laser beams converged on a single pellet of fuel



1) https://en.wikipedia.org/wiki/Inertial_confinement_fusion

Left: https://en.wikipedia.org/wiki/National_Ignition_Facility

Right: <https://lasers.llnl.gov/news/inside-look-advanced-radiographic-capability-arc-laser>

Progress over that 75 years?

In 2012 the **National Ignition Facility** program was officially discontinued ¹

having achieved only 1/10th of the conditions required to ignite Nuclear Fusion, ¹

after a net expenditure (estimated by the NIF itself) of **3.5 billion dollars** ²

Projects **have** demonstrated the occurrence of nuclear fusion, but none has yet even

achieved **breakeven** (where fusion power out = net power put into the reactor)

I could not even find a source **willing** to plot a history of fusion power out vs. in

Continuing work? The **International Thermonuclear Experimental reactor** (ITER) ³

Based in France, this cooperative project costing **20 billion dollars** ⁴

does not even have a breakthrough or game-changing idea in their sights

Indeed, contradicting some theoretical studies,⁵ ITER's plan is to just build a

BIGGER Tokamak reactor in the hopes of learning enough along the way,

that a viable fusion reactor might **eventually** be designed and built

1) <https://www.iter.org/newsline/-/3037> 2) <https://lasers.llnl.gov/about/faqs> 3) <https://en.wikipedia.org/wiki/ITER>

4) <https://www.sciencemag.org/news/2016/04/updated-panel-backs-iter-fusion-project-s-new-schedule-balks-cost>

5) <http://iopscience.iop.org/article/10.1088/0029-5515/55/3/033001/pdf>

To me as a scientist, 75 years of slow incremental progress strongly suggests:

That it's time to try something REALLY different!

And you **may** have heard a lot of buzz about new approaches to Fusion, for example:

- Lockheed Martin's secret mini reactor (that will fit on a truck!) ¹
- Lawrence Plasma Physics' proton-boron fusion reactor ²
- Helion energy's magnetic compression reactor ³
- General Fusion's liquid metal vortex shockwave reactor
- University of Washington's "Dynomak" variation on existing Tokomak

But none of these have been built (or there's zero public proof of this!)

Instead, many appear as yet to be little more than crowd-funding solicitations

And a few carry the distinct reek of cynical corporate image building

1) <http://www.lockheedmartin.com/us/products/compact-fusion.html>

2) <http://spectrum.ieee.org/energywise/energy/nuclear/how-far-can-crowdfunded-nuclear-fusion-go>

3) <http://spectrum.ieee.org/energywise/energy/nuclear/silicon-valley-goes-long-on-nuclear-fusion>

What about Cold Fusion (and the "scientific conspiracy" against it)?

For the full first half of my career, I worked in the basic research branch of Bell Laboratories

There I rubbed shoulders with a thousand or so fellow physicists, chemists, engineers . . .

In those 21 years, no other event so electrified us as the first rumors of cold fusion

When the unpublished manuscript announcing it began to circulate, anyone and everyone having relevant knowledge and appropriate laboratory equipment dropped whatever they had been researching, and began trying to replicate the reported results.

But by late in that week our shared lack of success was becoming more and more discouraging (especially given the relative simplicity of the experiments that had been reported).

Nevertheless, on the following Saturday night, attending a dinner party, some of us still risked irritating our spouses by repeatedly calling back into the lab, hoping to learn of a last minute breakthrough . . .

Yes, chemists' success at cold fusion would have embarrassed nuclear physicists

But the overwhelming majority of scientists were hoping (indeed dreaming) it was for real

Thus, in the end, we were just as disappointed as everyone else

WHEN might we achieve commercial scale Fusion Power?

On this note set's Resources webpage¹ I've collected predictions about Fusion's future:

The Path to Fusion Power

Fusion as a Future Power Source: Recent Achievements and Prospects

Nuclear Fusion - Do The Math

It's the 21st Century - Where's My Fusion Reactor?

Why Nuclear Fusion Is Always 30 Years Away

The Uncertain Future of Fusion Energy

Why Nuclear Fusion is Gaining Steam – Again

Some are opaquely technical, others surprisingly understandable (e.g., Do the Math)

A few offer bullish predictions of success, most are much more cautious

But nearly all put a *prototype fusion reactor at least 30 YEARS IN THE FUTURE*²

Add to that 20 YEARS for taking a prototype through full development & deployment

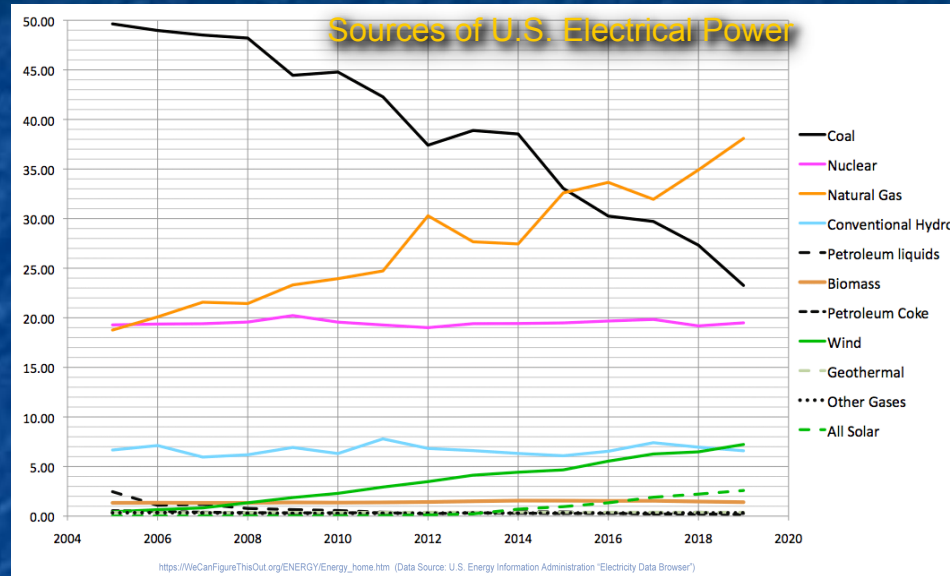
And you are almost certainly too late to counter catastrophic climate change

¹ Resources Webpage: https://wecanfigurethisout.org/ENERGY/Web_notes/Exotics/Exotics%20-%20Supporting.htm

² FYI: To my personal knowledge, Nuclear Fusion has been predicted to be "30 Years in the Future" since at least the 1960's

Leading to my personal takeaways from this note set:

I'll continue to pin my hopes on some of the technologies already in this figure:



Or on "exotic" (a.k.a. "emerging") technologies for which scientific calculation

confirms that the power source into which they hope to tap is large enough

that there's the **possibility** of someday qualifying for such a figure

At least if no "as yet unimaginable" breakthrough is required to capture that power

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This set of notes was authored by John C. Bean who also created all figures not explicitly credited above.

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