

CHAPTER 8

Development and analysis of vertical-axis wind turbines

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Vertical-axis wind turbines (VAWTs) have been demonstrated to be effective devices for extracting useful energy from the wind. VAWTs have been used to generate mechanical and electrical energy at a range of scales, from small-scale domestic applications through to large-scale electricity production for utilities. This chapter summarises the development of the main types of VAWT, including the Savonius, Darrieus and Giromill designs. A summary of the multiple-streamtube analysis of VAWTs is also provided to illustrate how the complex aerodynamics of these devices may be analysed using relatively straightforward techniques. Results from a double-multiple-streamtube analysis are used to illustrate the details of the performance of VAWTs in terms of turbine blade loads and rotor power output as a function of fundamental parameters such as tip speed ratio. The implications for VAWT design are discussed.

1 Introduction

Vertical-axis wind turbines (VAWTs) come in a wide and interesting variety of physical configurations and they involve a range of complex aerodynamic characteristics. Not only were VAWTs the first wind turbines to be developed but they have also been built and operated at a scale matching some of the biggest wind turbines ever made. VAWTs in principle can attain coefficients of performance, $C_{p,max}$, that are comparable to those for horizontal-axis wind turbines (HAWTs) and they have several potentially significant advantages over the HAWTs.

These advantages include the fact that VAWTs are cross-flow devices and therefore accept wind from any direction. Thus, in principle, they do not need a yaw mechanism to ensure that they are aligned to the wind as is the case with all



horizontal axis machines. Another key advantage is that the mechanical load may be connected directly to the VAWT rotor shaft and located at ground level. This removes the need for a substantial tower to support the weight of equipment such as the gearbox, generator and yaw mechanism. There is also no need for slip rings or flexible cables to connect the generator to the load, which can be an important point for small-scale turbines.

From the 1970s to the 1990s a number of research groups and companies developed and built hundreds of VAWTs and a great deal has been learnt from that experience. But despite the inherent advantages of VAWTs they have fallen significantly behind HAWTs in recent years in terms of technical development and in the size and number of units manufactured. This has occurred for a number of reasons, not least because of some inherent disadvantages of VAWTs.

As the VAWT blades rotate about the main rotor shaft the velocity of the air relative to the blade is constantly changing in respect of both magnitude and direction. In addition, each blade will interact with the wakes of other blades, and possibly its own wake, when it passes through the downstream half of its path about the turbine axis. Both these effects result in fluctuating aerodynamic forces on the blades, which in turn lead to a potentially significant fatigue issue for the design of the blades and overall turbine structure. The fluctuating blade loads also lead to a varying torque transferred to the mechanical load.

Many designs of VAWTs produce very low torque when they are stationary and may produce negative torque at low tip speed ratios, so they must be powered up to a speed at which the aerodynamic torque is sufficient to accelerate the rotor to normal operational speeds. A further disadvantage is that parasitic drag losses may be high for a given VAWT design. This situation can arise when the VAWT blades need to be mounted on structures (spars, beams, cables, etc.) that rotate with the blades or are located upstream of the blades. The drag forces on these passive components can lead to significant parasitic losses in respect to rotor torque and power output. This has inhibited the successful development of a number of VAWT designs.

Nevertheless there continues to be widespread interest in VAWTs as a means of generating electrical and mechanical energy from the wind. Novel VAWT turbine designs appear relatively frequently at the time of writing and a number of small companies appear to be undertaking development of VAWTs for small-scale application, particularly in respect to domestic dwellings.

2 Historical development of VAWTs

2.1 Early VAWT designs

VAWTs appear to have been developed long before their horizontal axis cousins. One of the reasons for this is that the VAWT has a number of inherent advantages including the fact that a drive shaft may be connected directly from the rotor to a mechanical load at ground level, eliminating the need for a gearbox. The early pioneers involved in the development of wind turbines many centuries ago applied VAWTs to the milling of grain, an application where the vertical axis of the millstone could be easily connected to the VAWT rotor. Quite a number of excellent



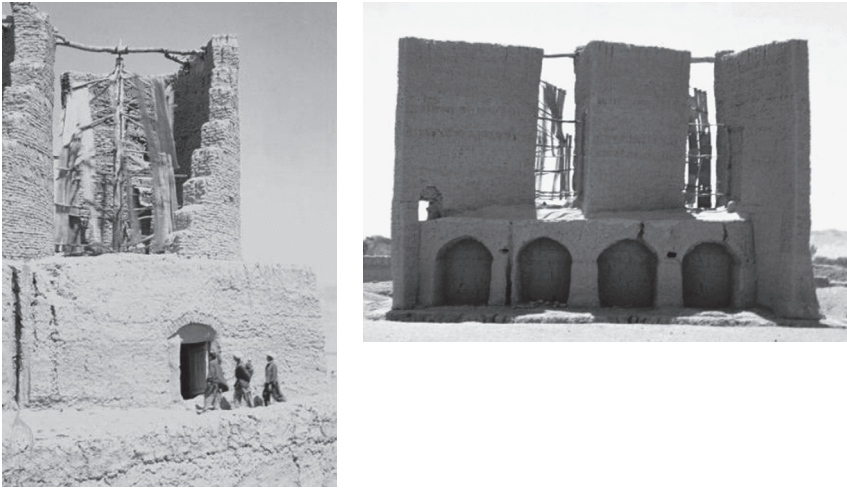


Figure 1: An example of VAWTs in the Sistan Basin in the border region of Iran and Afghanistan. Note in the right hand image how the upstream wall is used to expose only one half of the rotor to the wind (photographs taken in 1971 near Herat, Afghanistan, copyright: Alan Cookson).

review articles have been published in the past detailing the historical development of wind turbines of all types [1, 2]. Virtually all of these reviews suggest that the very earliest wind turbines were indeed VAWTs and it is thought that these were first used in Persia for milling grain more than 2000 years ago. These early wind turbines were essentially drag devices with a rotor comprising a number of bundles of reeds, or other simple blades, on a timber framework. The rotor was housed within a walled enclosure that channelled the flow of wind preferentially to one side of the rotor thereby generating the torque necessary to rotate the millstone. This type of device was still in use during the latter half of the 20th century and an example located in the border region of Afghanistan and Iran is shown in Fig. 1 [3].

The Persian and Sistan VAWTs had rigid vanes to generate torque whereas other designs have used sails that can effectively pitch with respect to their alignment on the rotor and thus can potentially increase efficiency. An example of a Chinese VAWT of the type used for many years for pumping applications, and which was described by King [4] for pumping brine for salt production, is illustrated in Fig. 2.

2.2 VAWT types

A wide variety of VAWTs have been proposed over the past few decades and a number of excellent bibliographies on VAWTs have been published that summarise research and development of these devices, including the survey by Abramovich [5]. Some of the more important types of rotor design are highlighted in the following sections.

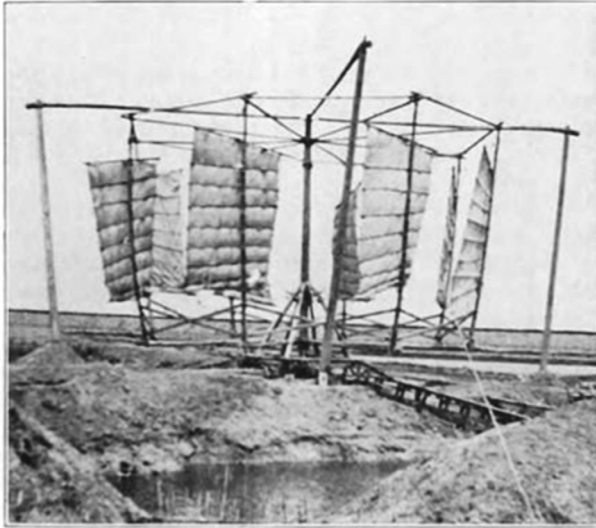


Figure 2: A Chinese VAWT used for pumping brine (photo taken in early 20th century) from King [4].

2.2.1 Savonius turbines

The need to pump water in rural/remote locations has long been a driver for the development of wind turbines. In the early 20th century a number of innovations were developed by inventors such as Savonius who patented his device in 1929 [6]. This utilised a rotor made from two half-cylinders held by a disc at each end of the rotor shaft, as shown in Fig. 3. The Savonius turbine has been popular with both professional and amateur wind turbine developers over the years, not least because of its simple and robust construction.

Many variations of the Savonius rotor have been developed and tested. However, because of the inherently high solidity and hence high mass of the Savonius turbine it has not been used for large-scale electricity production. Nevertheless, it continues to find favour in a number of areas of application, including building-integrated wind energy systems which are now attracting attention as building designers seek to reduce the ecological footprint of building structures and their operations. Müller *et al.* [3], for example, explore the potential of VAWTs installed on buildings. Figure 4 shows an example of this type of application where Savonius turbines are mounted on the natural ventilation stacks of the landmark Council House 2 (CH2) Building in Melbourne, Australia. The low tip speed of rotors such as the Savonius has a number of attractions, not least that they are likely to produce less aerodynamic noise, which is an important issue for turbines included as part of inhabited structures. However, a number of considerable challenges remain to be overcome before building-integrated wind turbines can provide a cost-effective means of generating electricity. These include the fact that the urban environment is characterised by low wind speeds and high turbulence.

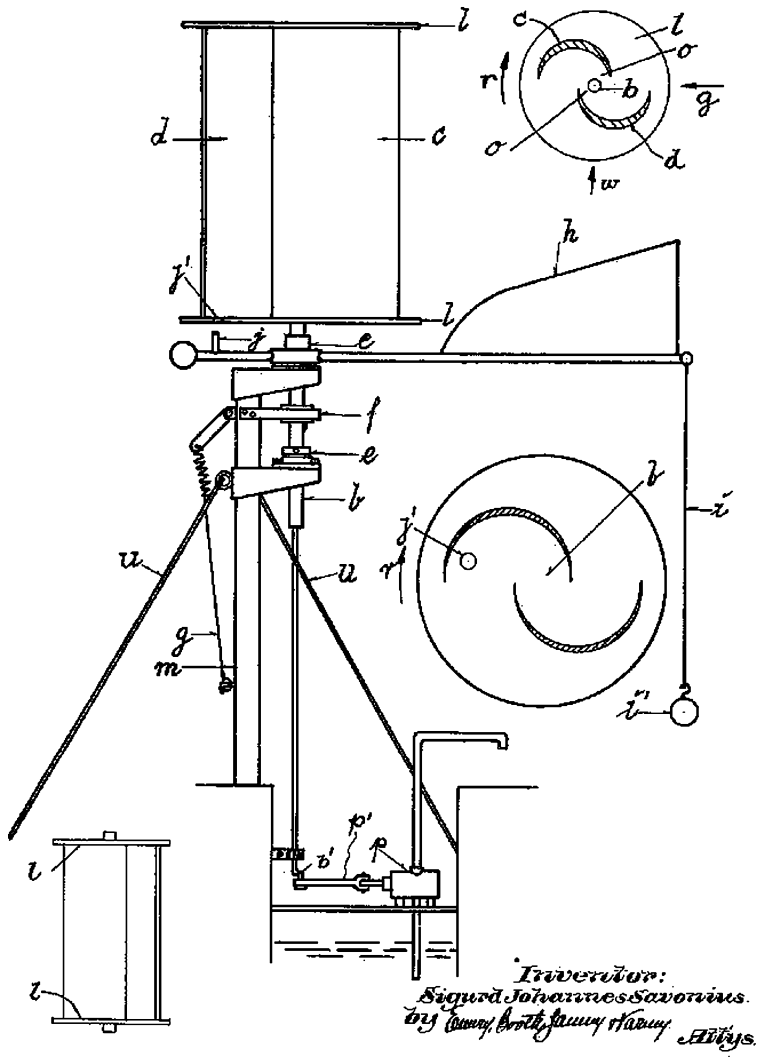


Figure 3: The principal embodiment of the Savonius VAWT patent [6].

The Savonius rotor is primarily a drag device with some inherent augmentation of the rotor performance available due to the air flow across each vane and mutual coupling of the two halves of the rotor. Like all drag machines it has a low operating tip speed ratio. This makes it less suitable for electricity generation than devices with higher tip speeds, since a high shaft speed is generally preferred to minimise the step-up ratio requirement of the gearbox coupling a rotor to a conventional electrical generator. Several new versions of the Savonius have been manufactured in recent years including devices with spiral blades of relatively short span mounted on a wide rotor hub.



Figure 4: Savonius turbines used to assist ventilation and generate electricity on the Council House 2 (CH2) landmark building in Melbourne, Australia (photographs – copyright Pauline Anastasiou).

There have been many studies of the performance of the Savonius rotor, however, it would appear that the coefficient of performance, C_p , is modest. Modi and Fernando [7], for example, tested a wide range of Savonius rotor geometries in a wind tunnel with variations in the degree of overlap and separation of the blades and rotor aspect ratio. Modi and Fernando also carried out important tests to determine the blockage effect of the turbine in the wind tunnel. Thus, they were able to extrapolate their results to estimate the performance of Savonius rotors under unconfined conditions. Their conclusion was that the best coefficient of performance of a geometrically optimised Savonius rotor was likely to be $C_{p,max} \sim 0.3$ at a tip speed ratio of $\lambda \sim 0.7$. Ushiyama and Nagai [8] carried out unconfined tests with various Savonius rotors located downstream of the exit of a wind tunnel. The maximum rotor coefficient of performance of $C_{p,max} \sim 0.23$ at a tip speed ratio of $\lambda \sim 1.0$ was found to be less than that reported by Modi and Fernando although it was not made clear whether the effects of bearing frictional losses were accounted for in the experiments. More recent studies have also been conducted on a number of geometric variations including stacking rotors one above the other and Rahai has reported on optimisation of the Savonius design using CFD analysis [9].

2.2.2 Darrieus turbines

In 1931 the invention by Darrieus [10] of his rotor with a high tip speed ratio opened up new opportunities for VAWTs in regards to electricity generation. The fundamental step forward made by Darrieus was to provide a means of raising the velocity of the VAWT blades significantly above the freestream wind velocity so that lift forces could be used to significantly improve the coefficient of performance of VAWTs over previous designs based primarily on drag. Darrieus also foresaw a number of embodiments of his fundamental idea that would be trialled at large scale many decades later. These included use of both curved-blade (Fig. 5a) and straight-blade versions of his rotor. He also proposed options for active control of the pitch of the blades relative to the rotor as a whole, so as to optimise the angle of attack

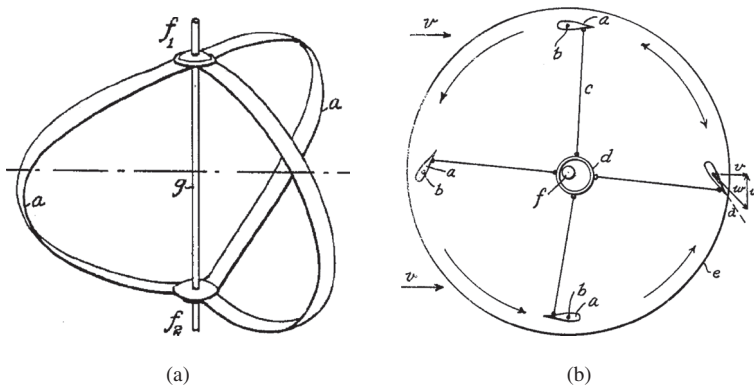


Figure 5: Images from the Darrieus VAWT patent [10]: (a) curved-blade rotor embodiment; (b) plan view of straight-blade rotor showing an optional active blade pitching mechanism.

of the wind on each blade throughout its travel around the rotor circumference (as shown in Fig. 5b). However, it was not until the energy crisis of the early 1970s that Darrieus' rotor was developed to the point whereby it became a commercially viable wind turbine.

The Darrieus turbine can take a number of forms but is most well known in the geometry sometimes called the “egg-beater” shown in Fig. 5a, where the two or three blades are curved so as to minimise the bending moments due to centrifugal forces acting on the blade. The shape of the curved blade is close to that taken by a skipping rope in the absence of gravity and is known as the Troposkein (“spinning rope”). The Sandia National Laboratory wind research team was one of the leading groups in development and analysis of the Darrieus curved-blade turbine. One of the first devices tested was the 2-m diameter Sandia research turbine, which was tested in a 4.6 m × 6.1 m wind tunnel and later field-tested at the Sandia National Laboratories wind turbine site [11]. The match between the wind tunnel and field tests was very close and this study indicated that a maximum power coefficient for this machine of $C_{p,max} \approx 0.32$ was achievable, which was promising for a relatively small device.

A second 5-m diameter demonstration turbine was then developed, also with a curved-blade rotor. This was superseded in 1975 by the 17-m diameter Sandia Demonstration Darrieus turbine which ran successfully over several years and provided important experimental information on the aerodynamics and structural dynamics of the Darrieus device. In 1988, a much larger 34-m diameter machine, known at the 34-meter Test Bed, was commissioned at Bushland, Texas. This device with aluminium blades had a rated power output of 0.5 MW and it provided a wealth of data on the field performance of a large VAWT [12]. Indeed the Sandia National Laboratories website [13] remains, at the time of writing, a rich source of information on many aspects of Darrieus turbines, including analysis, design and performance.

At least 600 commercial VAWTs were operating in California in the mid-1980s, the vast majority of these were Darrieus machines. The Flowind Corporation was probably the most successful manufacturer of VAWTs during this period and collaborated with Sandia Laboratories on the development of an enhanced Darrieus wind turbine [14] with superior aerodynamic and structural performance. By July 1995 Flowind were operating over 800 VAWTs in the Altamont and Tehachapi passes in California. However, the company's fortunes were to take a turn for the worse and they were bankrupt by 1997.

Canadian researchers also played a major part in the development of utility-scale Darrieus wind turbines. A recent book by Saulnier and Reid [15] details some of the pioneering work carried out by engineers of the Canadian Research Council (CNRC) and the Institut de Recherché d'Hydro-Québec (IREQ). The devices developed and tested included the 225 kW Darrieus turbine with a rotor 24 m in diameter and 36 m in height that was installed on the Magdalen Islands in the Gulf of St. Lawrence in 1977 and operated until 1983. A number of other research Darrieus turbines were also built and tested. A fully instrumented 50 kW Darrieus was constructed at IREQ in 1983 by Daf-Indal (Mississauga, Ontario) which was one of the series of commercial prototypes produced by the company and erected in many provinces of Canada under a program of the National Research Council of Canada [16]. However, the most significant Canadian VAWT project commenced in 1982 when IREQ, CNRC and other collaborators commenced work on the largest VAWT ever built. This was the curved-blade Darrieus Éole turbine rated at 4 MW with a two-bladed rotor, 96 m in height and 64 m in diameter (Fig. 6). The device operated successfully for over 30,000 hr during a 5-year

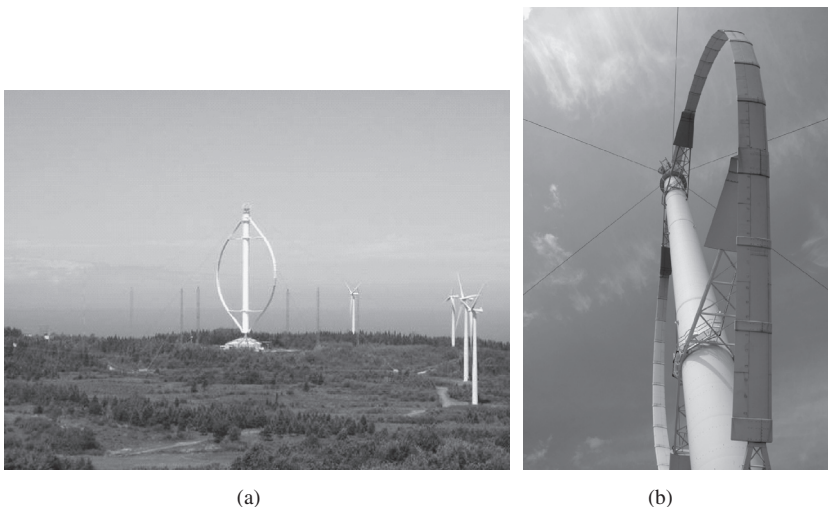


Figure 6: The world's largest VAWT, the Éole 4MW Darrieus turbine located at Cap-Chat, Quebec: (a) view of the turbine as part of the Le Nordais/Cap Chat Wind Farm [20]; (b) view of the rotor (photograph – copyright Alain Forcione).

period from March 1988 and produced over 12 GWh of electricity in the Le Nordais/Cap Chat wind farm (which also includes 73 NEG-Micon 750 kW HAWTs at the time of writing) [17].

One of the characteristics of the Darrieus family of turbines is that they have a limited self-starting capacity because there is often insufficient torque to overcome friction at start-up. This is largely because lift forces on the blades are small at low rotational speeds and for two-bladed machines in particular the torque generated is virtually the same for each of the stationary blades at start-up, irrespective of the rotor azimuth angle relative to the incident wind direction. Moreover, the blades of a Darrieus rotor are stalled for most azimuth angles at low tip speed ratios. As a result large commercial machines generally need to be run up to a sufficiently high tip speed for the rotor to accelerate in a given wind velocity. Nevertheless, two-bladed Darrieus machines do have the capacity to self-start, albeit on a somewhat unpredictable basis, and although this is advantageous in most circumstances it can cause problems. A case in point was in 1978 when the 225 kW Magdalen Islands Darrieus turbine was left for a few hours overnight without a brake engaged due to maintenance issues and the belief that such turbines would not self-start. The following morning researchers returned to find the turbine rotating at high speed with no load. As a result an energetic resonance developed in one of the rotor guy wires so that the guy came into catastrophic contact with the rotor which was entirely destroyed [17]. Another scenario where this can be a problem is when the turbine starts and turns initially at low tip speed ratio in a strong wind. If there is then a sudden decrease in the wind speed so the tip speed ratio increases the rotor power output may then be sufficient to cause rapid acceleration and damage may occur [18]. Self-starting capability may be enhanced through a number of strategies including: increasing solidity; using an odd number of blades; providing a form of blade pitch mechanism; and using blades that are skewed so that the blade azimuth angle is a function of axial distance along the rotor. A recent study of the self-starting characteristics of small Darrieus machines has been reported by Hill *et al.* [19].

In terms of noise generation there appears to have been very little experimental or theoretical analysis reported on Darrieus rotors or other types of VAWTs. Since Darrieus turbines have relatively low tip speeds compared to HAWTs one might expect the noise generated to be less problematic, as indicated by the analysis of Iida *et al.* [21].

2.2.3 Straight-blade VAWTs

The name Darrieus is usually associated with the curved-blade version of Darrieus' patent. However, a great deal of work over the past three decades has gone into the development and analysis of the straight-blade version of his original invention, which is sometimes known as the H-VAWT from the shape of the blades and supporting spars. One of the key researchers in the 1970s was Peter Musgrove who spent over 20 years working on wind turbines at the University of Reading.



Musgrove recognised that one of the key challenges facing VAWTs was the need to control the power output of the device at high wind speeds and that active pitch control of blades would result in an unnecessarily complex mechanical system for large devices. His research team developed a furling system whereby the straight blades could be hinged at their mid-point so that the angle of the blades relative to the axis of the rotor could be adjusted by mechanical actuators. A number of geometries were developed (including the V-VAWT machine and tested at small scales (e.g. diameter of order 3 m). However, it should be mentioned that the furling method described above can potentially lead to high transient vertical lift forces due to the effects of turbulence, which may in turn lead to high loads or failure of the supporting radial arms [18].

In the mid-1980s the UK Department of Energy supported the development of several VAWTs based on Musgrove's design. These were developed by a UK company VAWT Ltd. and several prototypes were built at the Carmarthen Bay test site of the Central Electricity Generating Board [22]. The first machine, the VAWT-450, was commissioned in 1986 and it had a 25-m diameter rotor with blades 18 m in length which provided a rated output of 130 kW at a wind speed of 11 m/s.

Subsequently a much larger version of this device was designed and built by VAWT Ltd. and also installed at Carmarthen Bay. The VAWT-850 had a 45-m diameter rotor and a rated output of 0.5 MW. The design did not include a furling system for the blades as previous experience with the VAWT-450 had demonstrated that this was unnecessary due to the inherent ability of the straight-bladed VAWT to avoid overspeed and excessive power generation through stall of the blades at high wind speeds. Although the VAWT-850 was a successful demonstration of the straight-bladed VAWT technology it suffered a catastrophic failure of one of the blades in 1991, apparently due to a manufacturing fault [22].

2.2.4 Giromills

One of the consequences of adopting a vertical axis for any wind turbine is that the apparent velocity of the wind at a particular location on a blade will change throughout each revolution of the rotor (Fig. 7). For example, when the blade is travelling upstream (i.e. when the azimuth angle $0^\circ < \beta < 180^\circ$) the resultant air velocity on the blade is greater than the tangential velocity of the blade relative to a stationary frame of reference, whereas, when the blade is travelling downstream ($180^\circ < \beta < 360^\circ$) the resultant wind speed is generally less than the tangential blade speed. This in turn means that the angle of attack on the blade is continually changing and is generally not optimal throughout its rotation about the axis of the turbine.

To improve this situation various means have been devised to optimise the blade pitch angle (i.e. the chord angle relative to a tangent to the path of the blade) as a function of azimuth angle, β . Systems have been devised to achieve this in a number of ways, including mechanical mechanisms with levers and/or pushrods connected between the blades and the main rotor shaft (as shown in Darrieus' original patent, Fig. 5b) or by means of aerodynamic mechanisms. Such turbines which seek to optimise the blade pitch angle are often known as Giromills [24], although some authors also refer to these systems as cycloturbines.



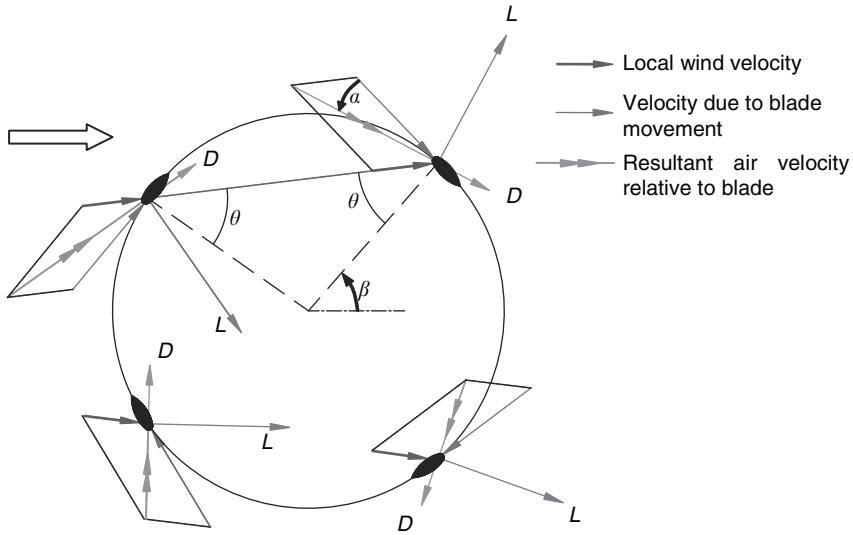


Figure 7: Plan view of velocity vectors and lift, L , and drag, D , vectors for a VAWT with freestream velocity, V_∞ , after Sharpe [23].

Kirke [25, 26] conducted an in-depth study of a number of three-bladed giromills with aerodynamic/mechanical activation of the blade pitch mechanism. These devices were referred to as “self-acting variable pitch VAWTs” with straight blades. Each blade was mounted at its mid-span on the end of the rotor radial arm and counterweighted so the mass centre coincided with the pivot axis, located forward of the aerodynamic centre. The pitch mechanism was activated by the moment of the aerodynamic force about a pivot, opposed by centripetal force acting on a “stabiliser mass” attached to the radial arm, such that the aerodynamic force overcomes the stabiliser moment and permits pitching before stall occurs. Lazauskas [27] had previously carried out modelling of three different types of blade pitch actuation mechanism and had predicted significant improvement in turbine performance compared to fixed pitch VAWTs. Wind tunnel tests reported by Kirke and Lazauskas [26] were carried out on a prototype rotor of 2-m diameter and solidity $Nc/R = 0.6$ with three blades each 1.0 m long and with 0.2 m chord and NACA0018 profile. Comparison between the wind tunnel tests and numerical results for various blade pitch scenarios were quantitatively good. Following these laboratory tests a 6-m diameter demonstration turbine was built by Kirke with the three blades being 2.5 m in length using the NACA0018 aerofoil profile (Fig. 8). Unfortunately, this device performed relatively poorly compared to theoretical predictions. This was apparently due to: (i) high levels of freestream turbulence at the test site, so that blades were subjected to variable wind speeds and the relatively massive rotor was slow to respond to sudden gusts and lulls and therefore operated well away from the optimum tip speed ratio most of the time; (ii) low wind speeds and therefore relatively low blade Reynolds number; (iii) high parasitic drag losses.



Figure 8: Three blade variable pitch VAWT developed by Kirke [25] clearly showing the counterweights incorporated in the blade pitch mechanism (photograph – copyright Brian Kirke).

2.2.5 Other designs and innovations

The field of wind energy systems has attracted many inventors and researchers over the years and in the field of VAWT innovation there has been a veritable plethora of designs put forward and, in many cases, demonstrated. However, there is little evidence in the academic literature that any of the relatively exotic designs will eventually be developed to the point of being competitive for electricity generation when compared with large, three-bladed HAWTs.

A class of VAWTs that have been investigated by a number of inventors include drag-based machines, or panemones [1]. One interesting device utilises a mechanically pitched blade which pitches through 180° during the course of one revolution of the rotor. The earliest technical paper related to this type of machine is thought by the author to be the description of the “Kirsten-Boeing Propellor” by Sachse [28], which was developed as a propeller for airships. Others have used the same principle to devise a wind turbine, rather than a propeller, and this is illustrated in Fig. 9 where it can be seen that the leading edge of a blade becomes the trailing edge on successive revolutions of the rotor. It can also be seen that such a device is strictly not a drag machine as lift may play a significant part in the development of the torque over a substantial fraction of the blade travel around the rotor.

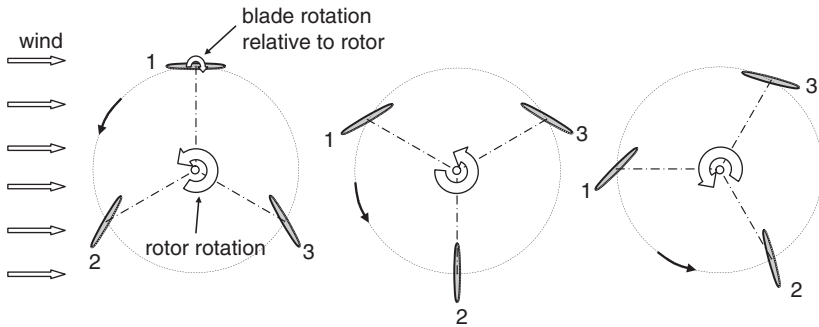


Figure 9: Principle of operation of a VAWT based on the variable pitch “Kirsten-Boeing Propellor” concept [28].

Theoretical modelling of this type of device indicates that the maximum coefficient of performance would be expected to be only $C_{p,max} \sim 0.2$ [29]. Like the Savonius rotor, this rotor does have the advantage of low tip speed ratio which will likely result in less noise and vibration issues, however, the high solidity ratio and hence material cost means that turbines of this type are highly unlikely to be a commercial success.

2.3 VAWTs in marine current applications

One of the hot topics in renewable energy at the time of writing is the development of marine current turbines (MCTs) to harvest the significant potential of tidal currents in various locations around the world. These devices are also known as hydrokinetic turbines, which include those operating on the same principles but in rivers and estuaries. Areas such as the English Channel and the north coast of Ireland have been identified as having great potential for this technology. The most common technology currently being applied in the field is that of horizontal axis MCTs such as that developed by Marine Current Turbines Ltd. [30] and Open-Hydro [31]. However, various research groups have investigated the feasibility of using vertical-axis MCTs which have obvious advantages in this application, particularly in that a yaw mechanism is not required to align the turbine with the wind [32–36].

3 Analysis of VAWT performance

As in the case of HAWTs, there are a number of levels of complexity with which one might analyse the performance of the VAWTs as outlined by authors including Touryan *et al.* [37], Strickland [38] and Wilson [24]. Allet *et al.* [39] classified the four main approaches to modelling of VAWTs as: (i) momentum models; (ii) vortex models; (iii) local circulation models; and (iv) viscous models, where the latter would include full viscous flow computational fluid dynamics (CFD)

models. Other bibliographic sources of information on analysis methods include those of Abramovich [5] and Islam *et al.* [40]. An extremely useful and relatively simple momentum model uses the actuator disc/blade element momentum theory (or strip theory) in a similar manner to the momentum model used for analysis of HAWTs. This type of analysis is outlined below and can be implemented relatively easily by anyone with a basic knowledge of fluid mechanics.

3.1 Double-multiple-streamtube analysis

The performance of a VAWT may be estimated using a momentum analysis at one of a number of levels of complexity [24]. The simplest approach is where a single streamtube is considered in which the interaction between the air flow and rotor is treated as a single actuator disc located on the axis of the rotor, perpendicular to the incident air flow. However, it is preferable to use a multiple-streamtube analysis since the resultant air velocities and forces acting on the blades are strong functions of the blade azimuth angle, β (Fig. 7). Strickland was one of the pioneers of this approach [41]. However, to more accurately represent the flow through the rotor, a VAWT may be represented as an “actuator cylinder”, whereby the cylinder surface is swept by the length of the rotating blades as shown schematically in Fig. 10. Air in the freestream interacts first with the upstream half of the cylinder and then the downstream half. A number of authors have described this double-multiple-streamtube methodology including Paraschivoiu [42–44] and Madsen [45]. Sharpe [23] in particular presented a very clear exposition of this method and his approach is summarised in the following section.

Using a methodology analogous to the actuator disc/momentum analysis for a HAWT one can define two velocity induction factors, a_u and a_d , which define the deceleration of the local air velocity relative to the freestream velocity, U_∞ , on the upstream and downstream faces of the cylinder, respectively. A number of

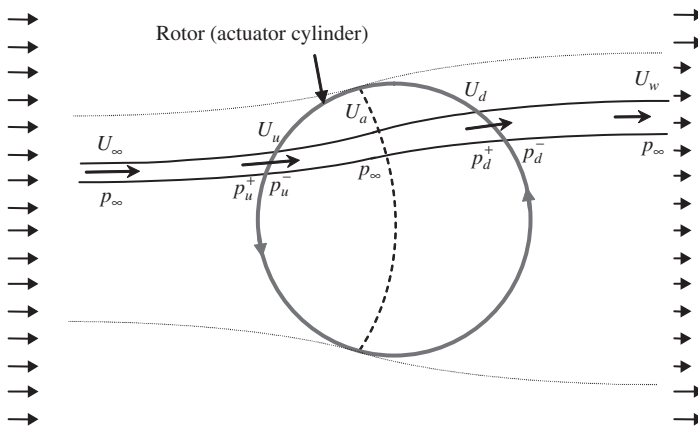


Figure 10: Plan view of a double-multiple-streamtube analysis of the flow through a VAWT rotor, after Sharpe [23].

streamtubes of rectangular cross-section are then followed from the upstream undisturbed flow to the upstream half of the cylinder where the flow in a particular streamtube is retarded to U_u such that:

$$U_u = U_\infty(1 - \alpha_u) \quad (1)$$

A local pressure drop from p_u^+ to p_u^- occurs across the upstream face of the actuator cylinder. Following the streamtube downstream the pressure recovers so that at a given point somewhere between the upstream and downstream faces of the cylinder the static pressure returns to the freestream value, p_∞ (Fig. 10). Here the local air velocity, U_a , is assumed to be

$$U_a = U_\infty(1 - 2a_u) \quad (2)$$

There is a further pressure drop at the downstream interaction with the actuator cylinder and the pressure then again recovers to the freestream datum some distance downstream in the wake which flows at velocity U_w :

$$U_d = U_a(1 - a_d) \quad \text{and} \quad U_w = U_a(1 - 2a_d) \quad (3)$$

The retardation of the flow through the domain of interest leads to an expansion in the cross-sectional areas of the streamtube, from A_u to A_d , through the rotor. The effect of this expansion is accounted for explicitly in the calculation of the magnitude and direction of the resultant wind direction at the turbine blades as illustrated schematically in Fig. 11.

In the case of VAWTs it is generally useful to resolve the lift and drag forces acting on the blades into components acting normally (radially) and tangentially (chordwise), F_n and F_t , respectively. In the case of a VAWT with blades of fixed pitch, the chord of the blades is generally held perpendicular to the radius from the rotor axis. The lift and drag coefficients, C_L and C_D , for a given aerofoil section may be manipulated to provide the non-dimensional normal

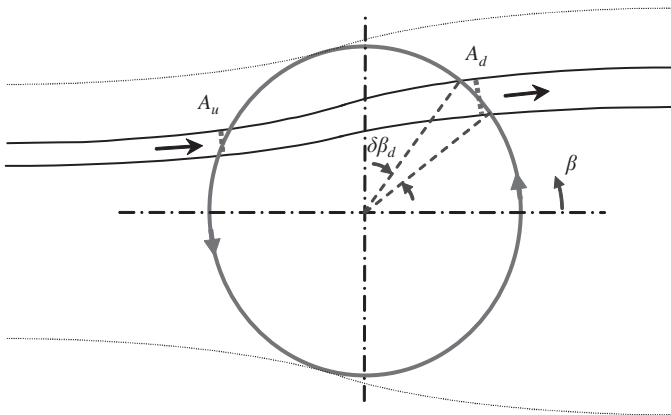


Figure 11: Illustration of streamtube expansion and nomenclature, after Sharpe [23].

and tangential coefficients, C_n and C_t , as a function of the angle of attack, α , as follows:

$$C_n = C_L \cos \alpha + C_D \sin \alpha, \quad C_t = C_L \sin \alpha - C_D \cos \alpha \quad (4)$$

The force acting on a blade in the direction of the local resultant air flow, W , is then:

$$F = \frac{\rho W^2}{2} c(C_n \cos \theta - C_t \sin \theta) \quad (5)$$

where ρ is the density of the air, W is the local resultant air velocity, c is the chord length and θ is the angle between the streamtube and the local radius to the rotor axis (see Fig. 7). Using the impulse-momentum principle the forces on the blade at the upstream and downstream portions of the cylinder may then be computed and related to the resulting deceleration of the flow to give expressions for the upstream and downstream induction factors:

$$a_u(1 - a_u) = \frac{Nc}{8\pi R} \frac{W_u^2}{U_\infty^2} \sec \theta (C_{nu} \cos \theta + C_{tu} \sin \theta),$$

$$a_d(1 - a_d) = \frac{Nc}{8\pi R} \frac{W_d^2}{U_\infty^2} \sec \theta (C_{nd} \cos \theta + C_{td} \sin \theta) \quad (6)$$

where N is the number of blades and R is the radius of the rotor. Equation (6) is solved iteratively together with the following auxiliary eqns (7) and (8) so as to find the unknown parameters in the problem. The angles of attack on the blade at the upstream and downstream locations are given by:

$$\tan \alpha_u = \frac{U_\infty(1 - a_u) \cos \theta}{\Omega R + U_\infty(1 - a_u) \sin \theta}, \quad \tan \alpha_d = \frac{U_a(1 - a_d) \cos \theta}{\Omega R + U_a(1 - a_d) \sin \theta} \quad (7)$$

where $\Omega = d\beta/dt$ is the angular velocity of the rotor. The local resultant relative velocities are then given by:

$$W_u = \sqrt{(\Omega R + U_\infty(1 - a_u) \sin \theta)^2 + (U_\infty(1 - a_u) \cos \theta)^2},$$

$$W_d = \sqrt{(\Omega R + U_a(1 - a_d) \sin \theta)^2 + (U_a(1 - a_d) \cos \theta)^2} \quad (8)$$

The torque, Q , generated by the blade for each of the streamtubes may then be estimated from:

$$Q_u = \frac{\rho W_u^2}{2} \frac{Nc}{2\pi R} A_u \sec \theta \left(C_{tu} \cos \theta + \frac{C_{nu}}{4} \right),$$

$$Q_d = \frac{\rho W_d^2}{2} \frac{Nc}{2\pi R} A_d \sec \theta \left(C_{td} \cos \theta + \frac{C_{nd}}{4} \right) \quad (9)$$

and hence the total torque and shaft power from the rotor may be determined by integration of eqn (9) around the circumference of the rotor.



The remaining task is to relate the forces acting on the blade and the torque generated to the local rotor azimuth angle, β . However, independent variables, such as W , a_u and a_d , have to this point been calculated only as a function of the angle θ between the streamtube and the local radius arm from the rotor axis. The blade azimuth angle β is related to θ through the degree of expansion of all the streamtubes passing through the rotor. It is a relatively simple matter to determine β after the local streamwise velocities, U_u and U_d , have been found as functions of θ and the corresponding azimuth angles may then be computed as detailed by Sharpe [23]:

$$\beta_d(\theta) = \int_0^\theta \frac{2U_u}{U_u + U_d} d\theta, \quad \beta_u(\theta) = \beta_d\left(\frac{\pi}{2}\right) + \int_\theta^{\pi/2} \frac{2U_d}{U_u + U_d} d\theta \quad (10)$$

The double-multiple-streamtube model described above has been implemented by the present author and illustrative results are presented in Figs 12–14 for a straight-bladed VAWT with a rotor radius of 20 m and blades based on the NACA0012 profile with a rotor solidity of $\sigma = Nc/R = 0.15$. Lift and drag data have been taken at an average blade Reynolds number of $Re_m = 1.0 \times 10^6$ from the data provided by a key publication on the lift and drag characteristics of aerofoils for VAWTs [46].

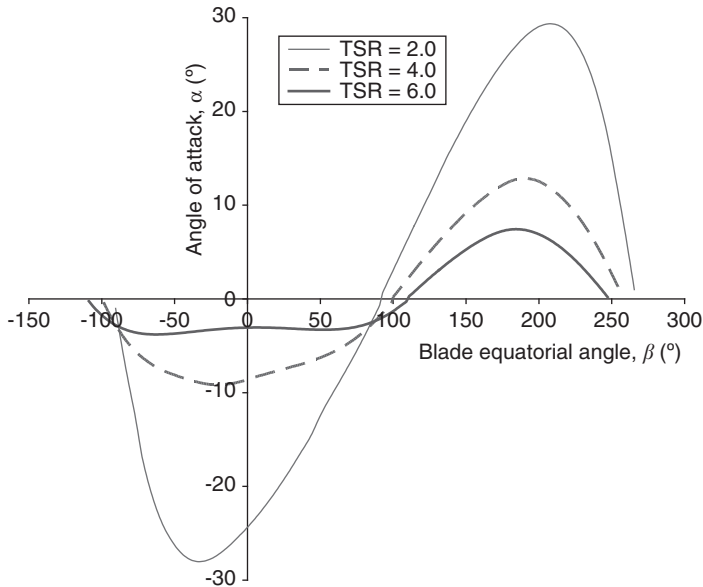


Figure 12: Angle of attack of resultant wind velocity, α , as a function of blade azimuth angle, β , and tip speed ratio. Predicted from a double-multiple-streamtube analysis of a straight-bladed VAWT (NACA0012H blade profile, $Re = 1.0 \times 10^6$, $R = 10$ m, $c = 0.5$ m, number of blades $N = 3$, $\Omega = 3.14$ rad/s).

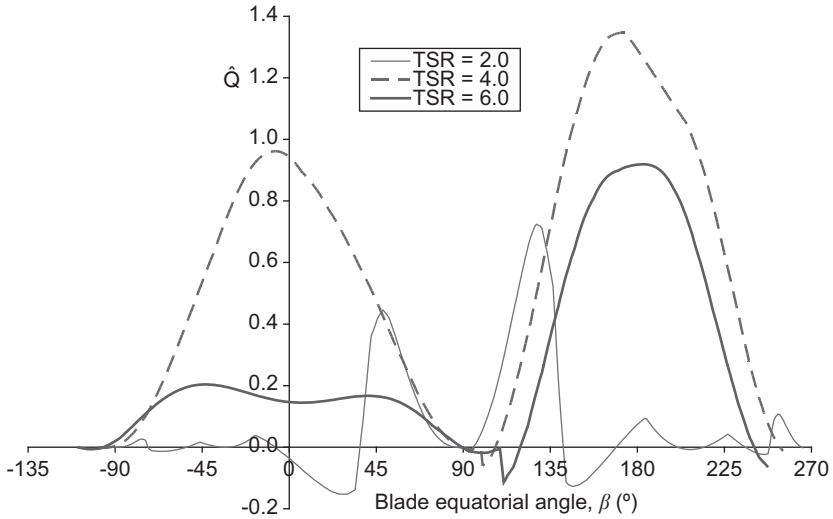


Figure 13: Non-dimensional torque coefficient, \hat{Q} , as a function of blade azimuth angle, β estimated from a double-multiple-streamtube blade element analysis of a VAWT (parameters as for Fig. 12).

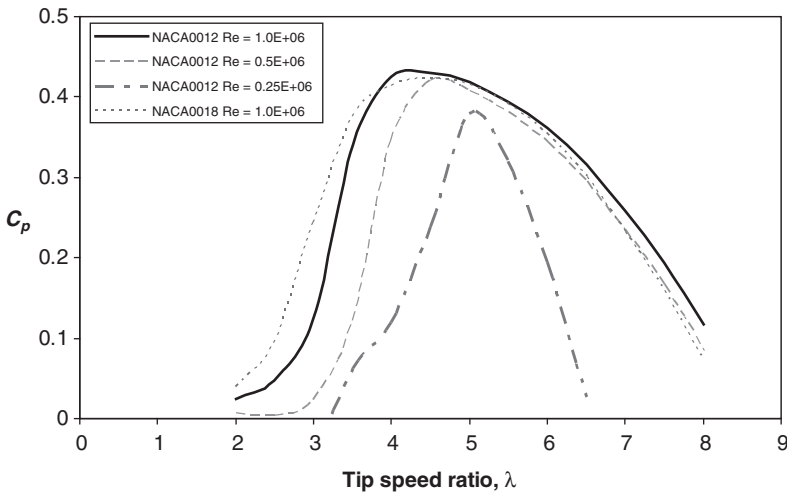


Figure 14: Double-multiple-streamtube results for the coefficient of performance, C_p , as a function of tip speed ratio, λ , for a straight-bladed VAWT turbine operating with different mean blade Reynolds numbers (solidity $cN/R = 0.15$, NACA0012 and 0018 profiles, with lift and drag data from [46]).

Figure 12 shows how the angle of attack on a blade varies throughout its travel about the rotor axis. The range of angle of attack is seen to decrease with increasing tip speed ratio. The non-dimensional torque per unit blade span generated at each azimuth angle, β , is shown in Fig. 13. This is a complex characteristic particularly at low tip speed ratios, λ . The complexity of the torque profile arises in part from the fact that NACA0012 blades will stall under steady-state flow conditions for any angle of attack, α , greater than approximately 14° . On the other hand, for the higher tip speed ratios of 4.0 and 6.0, the blade does not stall and there is a positive torque produced for the vast majority of azimuth angles. However, at a tip speed ratio of 2.0 the blades pass in or out of stall at four azimuth angles (-75° , 45° , 134° and 251°) and the blade is stalled for a very significant fraction of the total travel, which in turn results in limited overall torque generated and hence only a modest coefficient of performance, C_p . Although stalling of the blades in this way reduces C_p and causes significant fatigue loads, it does mean that an electrical generator connected to the rotor will benefit from passive overspeed protection at high wind speeds.

For the particular example chosen here with a mean blade Reynolds number of $Re_m = 1.0 \times 10^6$ the maximum coefficient of performance is $C_{p,max} \approx 0.43$ at an optimal tip speed ratio of $\lambda \approx 4$ as shown in Fig. 14. The performance of a VAWT rotor is strongly dependent on the blade Reynolds number as illustrated in Fig. 14 which serves to show that as the physical scale of a turbine is reduced so the maximum coefficient of performance decreases and the same is true of the range of tip speed ratios over which the turbine performs effectively. It could be said that VAWTs are particularly susceptible to reduction of C_p at low Reynolds numbers, since a lower Reynolds number limits the maximum lift coefficient that can be achieved with increasing angle of attack prior to stall. Thus, the effect of Reynolds number on the performance of small turbines may be more important for VAWTs as compared to HAWTs. It can also be seen that the thickness of the aerofoil has some influence on the C_p versus λ characteristic of the turbine (the NACA0012 aerofoil having a maximum thickness of 12% of the blade chord as opposed to 18% for the NACA0018).

The performance estimates from the double-multiple-streamtube methodology presented here do not account for a number of effects that may significantly reduce the output from a VAWT in practice. Parasitic drag loss is one of the key parameters that should be modelled by the designer of a VAWT. The loss of net power output due to the presence of components such as the radial arms on which the blades of a straight-bladed VAWT are mounted may be significant. Modelling such losses using a momentum model is relatively straightforward as the drag coefficients for beams and streamlined spars are well known [47]. These losses become increasingly important as the physical scale of the turbine is reduced. In addition, care should be taken in the interpretation of results from multiple-streamtube momentum models, particularly at high tip speed ratios, where large induction factors may be calculated which in turn lead to unrealistic wake velocity results.



3.1.1 Double-multiple-streamtube analysis of curved-blade VAWTs

The double-multiple-streamtube method described above considered a blade element located at a given radius from the rotor axis. This radius would be constant over the length of each blade for a straight-bladed VAWT. It is a relatively simple matter to adapt the equations above to other VAWTs such as curved-blade Darrieus machines where the blade radius is a function of elevation. Many curved-blade Darrieus machines have been constructed with a variant of the Troposkien blade shape where the ends of each blade comprise straight sections and the middle section has a constant radius. Whatever the actual shape of the blade may be the resultant velocity at a particular blade element is a function of the elevation from the mid-plane of the rotor (Fig. 15) and we can define $\zeta(z)$ as the angle of the blade element to the vertical.

Since only the wind velocity component normal (not spanwise) to the blade results in lift and drag forces, we require the magnitude of the local resultant velocity normal to the plane of the blade element, W , as illustrated in Fig. 15. W is then given by:

$$W = \sqrt{(\Omega r + U_\infty(1-a)\sin\theta)^2 + (U_\infty(1-a)\cos\theta\cos\zeta)^2} \tag{11}$$

where $r(z)$ is the local radius of the blade element. The aerodynamic forces acting on the blade element in the horizontal plane can be determined by modifying eqn (5) so as to account for angle ζ as follows [23]:

$$F = \frac{\rho W^2}{2} c(C_n \cos\theta \cos\zeta - C_t \sin\theta) \tag{12}$$

When analysing a curved-blade VAWT using the double-multiple-streamtube model, eqns (6) – (9) must also be modified so that C_n is replaced by $C_n \cos\zeta$, $\sec\theta$ is replaced by $\sec\theta \sec\zeta$ and the maximum rotor radius, R , is replaced by the local radius, r .

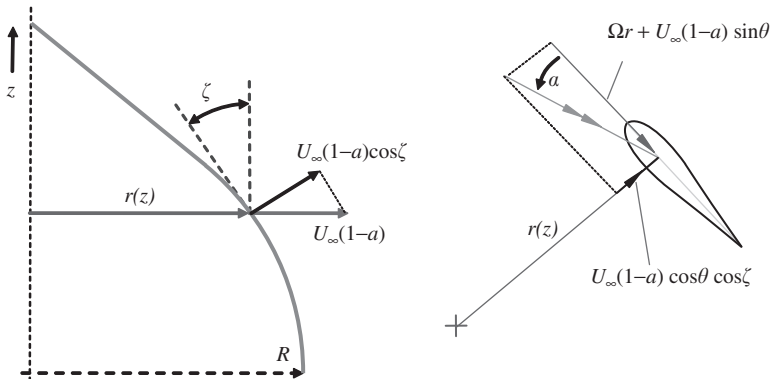


Figure 15: Schematic of kinematics of blade and wind velocity on a curved VAWT blade: (a) elevation of top half of blade showing component of wind acting normal to the plane of a blade element; (b) resultant velocities acting normal to the blade.

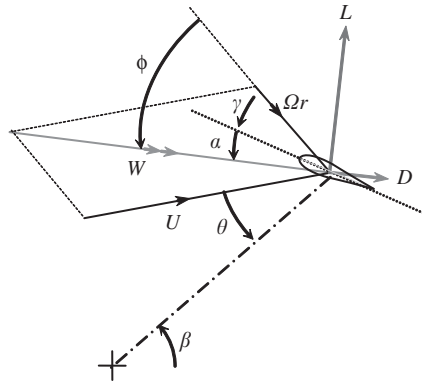


Figure 16: Illustration of the kinematics of a variable pitch VAWT blade.

3.1.2 Variable pitch VAWTs

In the double-multiple-streamtube model described above the blade pitch is held constant with respect to azimuth angle, β , i.e. the chord is perpendicular to the radius arm of the rotor. However, it is a relatively simple matter to modify the double-multiple-streamtube model to incorporate passive or active pitch control [27, 29]. In addition, one can also model the effect that some investigators have reported whereby an improvement in performance for fixed pitch turbines can be achieved if there is a slight *toe out* of the blades, as this reduces stall on the upstream pass. However, the resolution of the lift and drag forces into the appropriate tangential and normal components can be algebraically tedious because of the need to introduce new parameters for the blade pitch angle, γ , and resultant wind velocity angle, ϕ , as illustrated in Fig. 16.

3.1.3 Flow curvature and dynamic stall

The double-multiple-streamtube momentum model described above is a quasi-steady-state model which relies on the lift and drag characteristics of the aerofoils determined generally from steady-state wind tunnel tests or from inviscid or viscous numerical simulations. It follows that this model does not inherently capture a number of flow phenomena that occur in VAWTs in practice, for example, flow curvature and dynamic stall.

The issue of “flow curvature” relates to the fact that the apparent air motion relative to a blade of a VAWT has a curvature due to the rotation of the blade about the rotor axis. This in effect changes the apparent angle of attack on the blade and can be treated from a quasi-steady standpoint. The rate of pitching of the blade relative to the undisturbed flow is equal to the rotational velocity of the rotor, Ω . Sharpe [23] proposes a correction to the normal force coefficient, δC_n , based on thin aerofoil theory to account for the pitching of the blade such that $\delta C_n = (dC_L/d\alpha)(c/R)(\Omega R/W)/4$. An indication of the magnitude of this effect is provided by Wilson [24] using previous work carried out at the Sandia National Laboratories, which showed that flow curvature may result in an offset in the apparent

angle of attack on a VAWT blade of the order of 3° for $\lambda = 3.5$ and $c/R = 0.2$. Flow curvature has the effect of increasing the normal force on the blades on the upstream half of the actuator cylinder and decreasing this force on the downstream half [23].

The second important issue is what is commonly known as “dynamic stall”. This complex transient phenomenon arises because of the rapidly changing angle of attack on a VAWT blade. At low tip speed ratios a hysteresis effect arises whereby stall occurs at higher angles of attack, α , than for steady-state flow (when α is increasing). Subsequent reattachment of the flow is also delayed for decreasing α . A number of empirical and theoretical models have been developed by authors such as Allet *et al.* [39], Oler *et al.* [48], Major and Paraschivoiu [49] and Liu *et al.* [50]. These models may be incorporated into a double-multiple-streamtube analysis to improve the modelling of the transient effects of stall and also into more complex vortex models such as the Sandia codes [51]. More recently Ferreira *et al.* [52] have reported on a detailed flow visualisation study of the dynamic stall phenomenon.

3.1.4 Application and limitations of the double-multiple-streamtube method

The double-multiple-streamtube analysis described above is relatively straightforward and can provide quantitative results that are useful for optimisation of VAWT geometry in terms of fundamental parameters such as: operating tip speed ratio, blade profile, rotor solidity and aspect ratio. The model may also be used to estimate the forces on the blades which can then form the input to structural analysis and optimisation of the rotor. The accuracy of the double-multiple-streamtube model is comparable to that of more complex analysis methods. Sharpe [23], for example, showed that his prediction of $C_{p,max}$ for the Sandia 17-m diameter Darrieus turbine was within a few percent of the experimental results reported by Worstell [53]. Wilson [24] presents a comparison of the results from a number of double-multiple-streamtube analyses with the experimental data of Worstell [53] and very good agreement is seen for tip speed ratios, $\lambda > 3$.

It should be noted that care must be taken in the application of the momentum analysis methodology. In particular, it is possible for high induction factors to be predicted for VAWTs operating at high tip speed ratios which may lead to unrealistic wake flows. Corrections for some other flow phenomena not dealt with above, such as tip losses, can also be incorporated in the double-multiple-streamtube methodology.

3.2 Other methods of VAWT analysis

Inviscid flow models have been used by many of the key researchers in the field of VAWT analysis in years past and this approach has been summarised in the key overview article by Wilson [24]. While fixed wake models are relatively straightforward to implement, free vortex simulations are extremely complex and costly in terms of computer processing time. Nevertheless the free vortex model methodology is accepted to be the most comprehensive and accurate method of modelling VAWTs [51]. This methodology has also been recently applied to the analysis of vertical-axis marine current turbine by Li and Calisal [54]. CFD codes have now



developed to the point where this viscous flow analysis tool is available to most researchers in the academic and commercial sectors. However, application of this tool to VAWTs is not straightforward as full transient analysis and significant mesh refinement are necessary for meaningful results. CFD analysis of VAWTs does not appear to have been widely reported in the literature to date, although the research team at the École Polytechnique de Montréal have previously reported on their development of several viscous analysis codes for VAWTs [39] and more recently articles have appeared on CFD analysis of vertical-axis marine current turbines [33, 34].

4 Summary

This chapter has summarised the principles of operation and the historical development of the main types of VAWT. The Darrieus turbine remains the most promising of the vertical-axis rotor types for application to the utility-scale generation of electricity. The intense period of research, development and demonstration during the 1970s and 1980s did not lead to the development of a technology that is able to compete commercially with the three-bladed HAWTs that have come to dominate the market at large scale. Nevertheless new opportunities are opening up in the areas of marine current turbines and building-integrated wind turbines where the VAWTs may yet be competitive. In principle, the aerodynamic analysis of VAWTs is more complicated than that of HAWTs due to the significant variation of air velocity as a function of blade azimuth angle. The double-multiple-streamtube analysis summarised herein provides a tool that is relatively straightforward to use for those wishing to undertake an analysis of conventional VAWT designs.

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