Using the program

1. **Home Page of the *WindPower* program.**
2. Return on investment, payback period and cost per kilowatt-hour.
3. Wind turbine power output profile including the zero-power output period.
4. Comparison with field data for a large wind turbine - the Vestas V80 2MW and V90 3MW turbines.
5. Smaller wind turbines - the Britwind (Evance) R9000, Skystream, Bergey Excel and Honeywell RT6000.
6. Comparison with field data for small wind turbines -the Skystream, Fortis Montana and the Turby VAWT.
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PelaFlow Consulting

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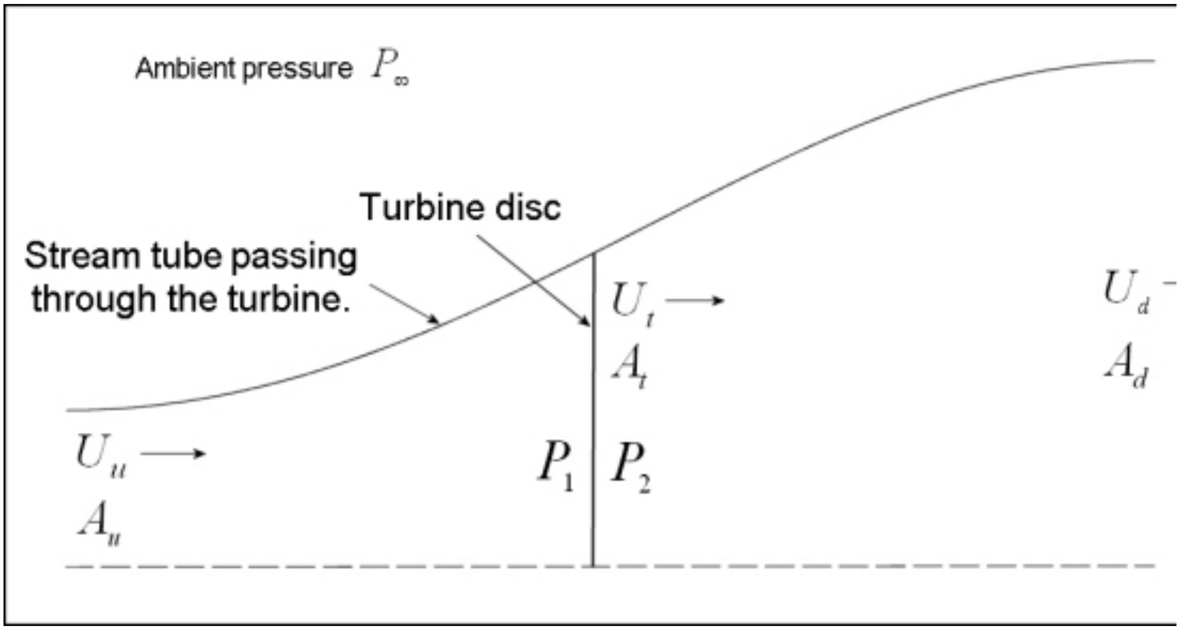
17. The Betz limit - and the maximum efficiency for horizontal axis wind turbines.
Can it be exceeded and does it apply to vertical axis wind turbines?

The simplest model of a wind turbine is the so-called actuator disc model where the turbine is replaced by a circular disc through which the airstream flows with a velocity U_t and across which there is a pressure drop from P_1 to P_2 as shown in the sketch. At the outset, it is important to stress that the actuator disc theory is useful (as will be shown) in discussing overall efficiencies of turbines but it does not help at all with how to design the turbine blades to achieve a desired performance.



The power developed by the wind turbine is

$$Power = (P_1 - P_2) A_t U_t$$

where A_t is the turbine disc area. Volume flow continuity gives

$$A_u U_u = A_d U_d = A_t U_t$$

From momentum conservation, the force exerted on the turbine is equal to the momentum change between the flow far upstream of the disc to the flow far downstream of the disc. Thus

$$(P_1 - P_2) A_t = Mass\ flow \times Velocity\ difference = \rho A_u U_u (U_u - U_d)$$

14. Wind turbine characteristics
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10. DOWNLOAD PAGE.
(1) Free WindPower trial program
(2) Buy full WindPower program
(3) Free turbine database
(4) Buy UK Wind Speed Database program

The final basic equations are Bernoulli's equation applied upstream and downstream of the actuator disc

$$P_\infty + \frac{1}{2}\rho U_u^2 = P_1 + \frac{1}{2}\rho U_t^2$$

$$P_\infty + \frac{1}{2}\rho U_d^2 = P_2 + \frac{1}{2}\rho U_t^2$$

where P_∞ is the ambient pressure in the flow both far upstream and far downstream of the actuator disc.
From equations (4a),(4b), (3) and (2)

$$\left(P_1 - P_2\right) = \frac{1}{2}\rho\left(U_u^2 - U_d^2\right) = \rho\frac{A_u}{A_t}U_u(U_u - U_d) = \rho U_t(U_u - U_d)$$

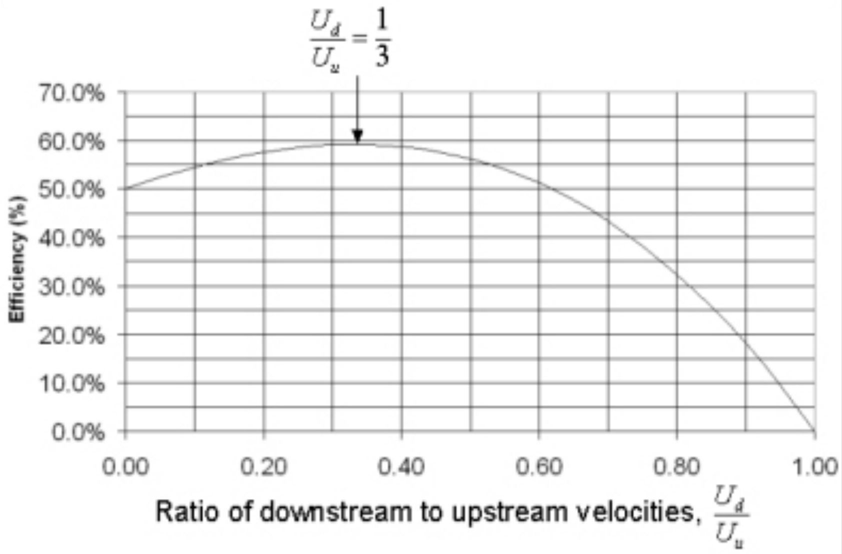
whence

$$\boxed{U_t = \frac{1}{2}\left(U_u + U_d\right)}$$

i.e. the velocity through the actuator disc is the mean of the upstream and downstream velocities in the stream tube.
Finally, from equations (1), (5) and (3), the efficiency is given by

$$\boxed{\eta = \frac{Power}{\frac{1}{2}\rho A_t U_u^3} = \frac{1}{2}\left(1 - \frac{U_d}{U_u}\right)\left(1 + \frac{U_d}{U_u}\right)^2}$$

The figure below shows the variation of efficiency (often referred to as the power coefficient, cp) with the ratio of downstream to upstream velocity. By differentiating equation (7), it is easy to show that the maximum efficiency oocurs when $U_d/U_u=1/3$ (i.e. when $A_d/A_u=3$). The efficiency is then $\eta=16/27 \approx 59\%$. This is the maximum achievable efficiency of a wind turbine and is known as the Betz limit - after Albert Betz who published this result in 1920. There are assumptions in the above anlysis such as the neglect of radial flow at the actuator disc but these have only a small effect on the final limiting result.



Variation of efficiency with the ratio of downstream to upstream velocity

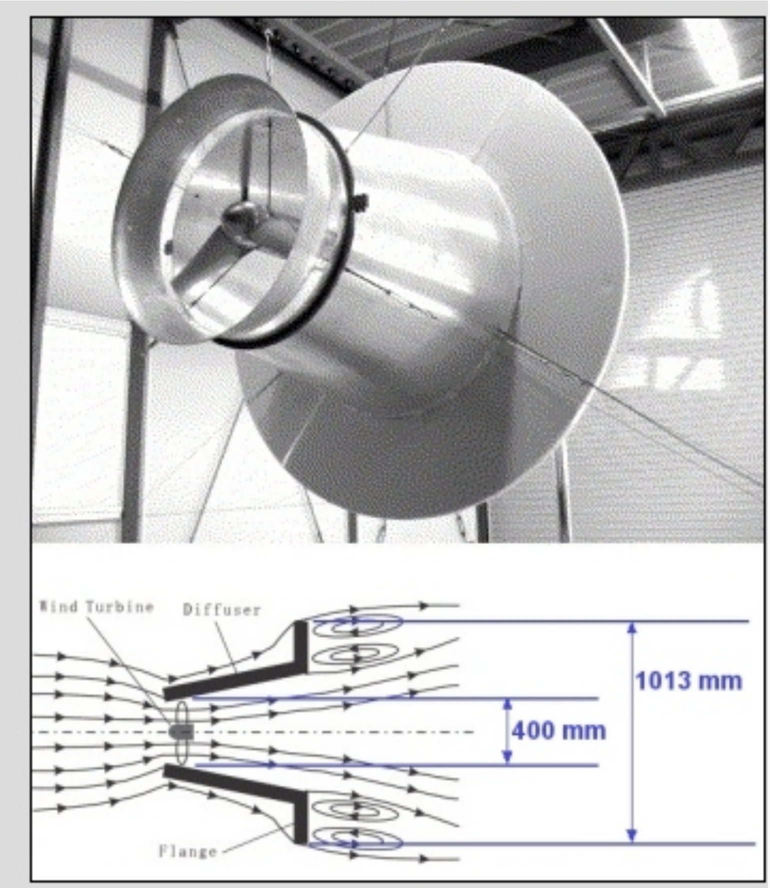
The point to note here is that as you reduce the downstream velocity in the expectation of increasing the power extracted from the wind, the area of the upstream stream tube that passes through the turbine reduces in size. In the limit as the downstream velocity is reduced to zero, the area of the upstream stream tube that passes through the turbine is just half the turbine area and the efficiency is thus 50%.

Can the Betz limit be exceeded for horizontal axis wind turbines?

It is important to note that the equations leading up to the Betz limit represent an overall momentum balance argument and therefore the argument still applies to any horizontal axis 'device' that replaces the actuator disc in the above derivation. The only question is what is the effective diameter of the stream tube that is influenced by the device? There have been numerous devices that claim to improve the efficiency of a wind turbine and the shouded turbine shown on the right is rather typical of these designs. In these 'shrouded' turbines, the general idea seems to be to use the shroud to create a low pressure region downstream of the turbine and thus draw more air through the turbine.

Generally, with these designs, there is little in the way of experimental data to support the efficiency claims but an exception to this seems to be some experimental and theoretical work carried out mainly at Kyushu University in Japan. The reference is given below. The figure on the right shows this design with a layout sketch.

In their report, the authors measure the turbine efficiency and, from a graph, they show a peak value of about 29% for the turbine on its own and a figure of about 110% with the shroud in place. In both cases, the efficiency or power factor is based on the swept rotor area. However, as can be seen from the sketch, the ratio of the shroud diameter to the rotor diameter is about 2.53 (i.e. 1013/400) and, if we base the efficiency of the shrouded turbine on the shroud cross-sectional area, the peak efficiency falls to 17%. In other words, a straightforward turbine with the diameter of the shroud would perform better in terms of efficiency than the shrouded turbine. The point here is that the Betz derivation still applies but the diameter of the stream tube influenced by the shrouded



turbine is closer to the shroud diameter than it is to the turbine diameter. This seems a fairly obvious conclusion and emphasises the point that there is no way of getting round the overall momentum balance between far upstream and far downstream in the derivation of the Betz limit. Moreover, in the case of the shrouded turbine, the drag on the shroud contributes nothing to the turbine power.

Reference

Abea K., Nishidab M., Sakuraia A. et al (2005) *Experimental and numerical investigations of flow fields behind a small wind turbine with a flanged diffuser.* Journal of Wind Engineering and Industrial Aerodynamics Volume 93, Issue 12, December 2005, Pages 951–970.

FloDesign-Ogin shrouded wind turbine.

An example of a shrouded wind turbine that is being put forward as a practical design is one that is described on the website oginenergy.com but which was originally designed by a company called FloDesign based in Massachusetts. A quite large demonstrator unit was put up in 2011 at Deer Island in Massachusetts as shown in the figure. It was a design rated at 100 kilowatts. The astonishing thing about this design is that it seems to have attracted multi-million dollar investments and yet nowhere in the web pages or downloadable literature is there any apparent awareness of the instrinsic limitations imposed by the arguments behind the Betz limit. It is almost certainly the case that the design will be less efficient than a conventional design whose rotor diameter is the same as the shroud diameter. It is difficult to see any advantages in designs like this and it is significant that no tests results on the turbine have been published which meet the IEC 61400-12 standards.

Does the Betz limit apply to vertical axis wind turbines?

The short answer to this question is 'No' although it is not obvious how to produce an equivalent theorem for a VAWT. The arguments that are used to derive the Betz limit for a HAWT do not apply directly to a vertical-axis wind turbine.

It is possible that an equivalent theorem can be produced by splitting the approaching stream tube into two parts; one passing through the advancing blades and the other passing through the retreating blades. The torque exerted on the VAWT will have to be matched by an equal and opposite angular momentum in the stream far downstream. It is much less obvious how to set up all the conservation relationships for a VAWT than a HAWT but it would make a good student or even a good post-graduate project.

From an experimental point of view, the efficiencies of VAWTs based on their frontal area seem always to be lower than a HAWT of equivalent frontal area and no VAWT has yet been tested to IEC61400-12 standards that has efficiencies in the upper range of large HAWTs - which can be in the region of 45%.

In spite of their lower efficiency, there are situations where a VAWT might be preferable to a HAWT (i.e. a gusty urban environment or some location with severe space constraints).

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